Algebra for Educators
E Term 2017
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MME529 Homework 1

Due Date: Tuesday May 9th, 2017

BASIC RULES FOR ASSIGNMENTS

I) Each student must compose his/her assignments independently. However, rough work may be done in groups;

II) Write legibly and use only one side of each sheet of paper;

III) Write at a level that will be clear to a classmate; there is no need to justify arithmetic or review definitions covered in class, but explanations should be given for anything that is not obvious. Write in full sentences.

IV) In general, late assignments will not be accepted for credit.

In a separate message, I will give you three prime numbers p, q and r. Your task is to search for pairs of consecutive numbers whose factorization into primes involves only these three. So you are considering the infinite set

$$S = \left\{ p^a q^b r^c \;\mid\; a, b, c \geq 0 \right\}$$

and you would like to discover all possible natural numbers n such that both n and $n+1$ belong to set S.
Write 2-4 pages on what you discover. Begin your essay with a reminder as to what your personal values of p, q and r happen to be. Then start exploring the set of numbers you can build by multiplying these together (with repetition). You are not likely to fully solve the problem. But you will certainly discover some examples, especially if you search using Maple . More importantly, you will discover some theorems and I want you to tell us about them.

Example: Suppose your given values are $p = 2$, $q = 3$ and $r = 5$. [I guarantee that your three primes will be different from these.] Then the set of natural numbers you are to study is

$$S = \{2^a 3^b 5^c \mid a, b, c \geq 0\} = \{1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18, 20, \ldots\}.$$

You might first consider the case where the exponents a, b and c are all positive. But if $n = 2^a 3^b 5^c$ with $a > 0$ and $c > 0$, then n is a multiple of ten, so it ends in zero. That means that $n+1$ must have last digit one, so $n+1$ cannot be even and cannot be a multiple of five. That is, $n+1$ must have form 3^k. An example is $n = 80 = 2^4 5^1$ with $n+1 = 3^4$. If you had these values of p, q, r and found nothing larger, you might report that $n = 80$ is the largest one you found of this type (where n factors into twos and fives while $n+1$ is a power of three). You might be able to prove that 2, 8 and 80 are the only natural numbers of this form, but don't worry about proving everything that you discover; conjectures will do just fine.

Another special case you might consider is where $n = 2^a$ and $n+1 = 5^c$. In this case, you would notice that n must have last digit 2, 4, 8 or 6 while $n+1$ has last digit five. So you can deduce that the exponent a must be an even number not divisible by four: $a \equiv 2 \pmod 4$. Looking at the last two digits might tell you more, or maybe you'll reduce both numbers mod eight. There are many ways to approach this.

Below, I include some MAPLE code which might help you explore.

Algorithm 1 List consecutive pairs in a sample of numbers with restricted prime factors

```
p := 2;   q := 3;   r := 5;
UpLimit := 20;
GoodNums := { seq( seq( seq( p^a*q^b*r^c , c=0..UpLimit), b= 0..UpLimit),
a=0..2*UpLimit) };
for n in GoodNums
do
if member( n+1 , GoodNums )  then
   printf(` n = %d n+1 = %d with factorizations \n`,n,n+1);
   print( ifactor(n), ifactor(n+1));
   fi;
od;
```

On the next page, I include a few notes and a few caveats about this MAPLE code.

Being a newcomer to MAPLE — or any software — can be extremely frustrating. So Rule #1 is: don't let it get to you. If a few attempts yield nothing useful, just swear at the computer and ask someone else for help.

For my own work, I use an ASCII interface for MAPLE and I get frustrated with their fancy new GUI that tries to typeset things as you enter them. This "user-friendly" interface has some unintended consequences such as rejecting correct code with line breaks in it. I think that you have to "begin" an "execution group" in the menu bar and then "end" it to get multi-line code like that included above to run.

The upshot of this is that I am not sure if a straight drag and drop of the code above into your MAPLE session will work. It does for me, but my simple MAPLE interface is nothing compared to the fancy one on PCs.

The carat ˆ doesn't look right on this page, so I used a wedge $\wedge$ to make the symbol larger when exponentiation is needed. A **for** loop in MAPLE has form

for *values in a set* **do** *list of commands* **od;**

and most examples start with something like

for n from 1 to 50 **do**

or

for n to 50 **do**

since the default starting point is always 1. But here we have built a set `GoodNums` and we instead iterate over all natural numbers n in that set.

Just as the code executed in a **for** loop is sandwiched between a **do** and an **od**, the MAPLE syntax for *if ... then ... else ...* is **if** ... **then** ... **else** ... **fi;** where the **else** branch is optional.