MA502 Linear Algebra
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MA502 Assignment 5

DUE DATE: Thursday November 16, 2017, at the beginning of class.

Please carefully read and observe the presentation rules for assignments in this course.

1(a) Suppose A is a 6×6 matrix with complex entries whose Jordan canonical form is

$$J = \left[\begin{array}{ccc|cc|c} 3 & 1 & 0 & 0 & 0 & 0 \\ 0 & 3 & 1 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & -4 \end{array}\right].$$

Find the Jordan canonical form for A^2. Explain.

2.) Define $\tau \in \mathcal{L}(\mathbb{F}^4)$ via $\tau : (x_1, x_2, x_3, x_4) \mapsto (0, x + 2 + x_4, x_3, x_4)$.

(a) Determine the minimal polynomial of τ. (Do not use a computer.) *[HINT: Show that τ and τ^2 have the same kernel.]*

(b) Determine the characteristic polynomial of τ. (Ditto.)

(c) Determine the Jordan canonical form for τ.

3.) Prove that any $n \times n$ matrix A with complex entries is similar to its transpose.

4.) Classify, up to similarity, all $n \times n$ matrices A with complex entries satisfying $A^3 = I$.

5.) τ be a linear operator on the complex vector space V of dimension n with eigenvalue λ. Let $\mathcal{C}_1, \ldots, \mathcal{C}_k$ be cycles of generalized eigenvectors for τ corresponding to eigenvalue λ. Let z_j be the initial vector of $\mathcal{C}_j$. Prove: *If $\{z_1, \ldots, z_k\}$ is a linearly independent set, then $\mathcal{C}_1 \cup \cdots \cup \mathcal{C}_k$ is also linearly independent.*