MA502 Linear Algebra
W. J. Martin
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MA502 Assignment 4

DUE DATE: Tuesday November 7, 2017, at the beginning of class.

Please carefully read and observe the presentation rules for assignments in this course.

1.) Use elementary row operations to compute the determinants of the following matrices over their respective fields. You may use the fact that the determinant of an upper-triangular matrix is the product of its diagonal entries.

$$(a)\ \ \mathbb{F}=\mathbb{C},\ \ A=\begin{bmatrix} i & i & i \\ 1 & -1 & 0 \\ -i & 1 & 0 \end{bmatrix}, \qquad (b)\ \ \mathbb{F}=\mathbb{Q},\ \ B=\begin{bmatrix} 0 & 0 & 0 & \frac{1}{2} & -\frac{1}{5} \\ 1 & -\frac{1}{2} & 0 & 2 & 0 \\ 1 & 1 & 0 & 0 & \frac{1}{4} \\ 2 & -1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$(c)\ \ \mathbb{F}=\mathbb{F}_5,\ \ C=\begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix}$$

(In part (c), we are performing arithmetic modulo five. E.g., $2 \cdot 3 = 1$.)

2.) For each of the examples in Problem 1, decide if the matrix is diagonalizable or not (over the given field $\mathbb{F}$). There is no need to give a basis of eigenvectors, but justify your answer in each case.

3.) Let $A = [a_{ij}]$ be an $n \times n$ upper-triangular matrix (so $i > j$ implies $a_{ij} = 0$). Prove that the characteristic polynomial of A is

$$\chi_A(z) = (z - a_{11})(z - a_{22}) \cdots (z - a_{nn}).$$

4.) Complete Problem 10.10 on page 249 in Hobart's notes.

5.) Complete Problem 10.15 on page 249 in Hobart's notes.