

MA502 Assignment 2

DUE DATE: Thursday, September 21, 2017, at the beginning of class.

Please carefully read the presentation rules below. **Any paper submitted which is sloppy or uses two sides of a page will be returned immediately with no credit.**

- 1.) Exercise 1 on p83 (Chapter 2) of Roman.
- 2.) Exercise 9 on p83 (Chapter 2) of Roman.
- 3.) Exercise 13 on p84 (Chapter 2) of Roman.
- 4.) Consider the operator τ on $\mathcal{C}^\infty(\mathbb{R})$ defined by $\tau : f(x) \mapsto f'(x)$ (first derivative). Describe all eigenvectors of operator τ . Explain.
- 5(a) Construct an example $A \in \mathcal{M}_{3 \times 3}(\mathbb{C})$ with exactly one eigenvector up to equivalence under scalar multiplication. Choose A so that the corresponding eigenvalue is i .
- (b) Construct, or prove non-existence of $A \in \mathcal{M}_{3 \times 3}(\mathbb{C})$ with **no** eigenvectors.
- (c) In part (a), what are the other eigenvalues of your matrix A ? Why do I know them already?
- 6.) Write each of the following matrices as a linear combination of projection matrices:

$$(a) \quad A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \quad (b) \quad B = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \quad (c) \quad C = \begin{bmatrix} 1 & 2 & 2 & 3 \\ 2 & 1 & 3 & 2 \\ 2 & 3 & 1 & 2 \\ 3 & 2 & 2 & 1 \end{bmatrix}$$

- 7.) Suppose P and Q are $n \times n$ projection matrices with real entries such that $PQ = 0$. What can you say about their column spaces? Explain.
- 8.) Let τ be a linear operator on vector space V over \mathbb{F} and suppose $\mathbf{v}_1, \dots, \mathbf{v}_k$ are eigenvectors belonging to distinct eigenvalues: $\tau \mathbf{v}_j = \lambda_j \mathbf{v}_j$ with $\lambda_j \neq \lambda_\ell$ unless $j = \ell$. Prove that the set $\mathcal{S} = \{\mathbf{v}_1, \dots, \mathbf{v}_k\}$ is linearly independent in V .

BASIC RULES FOR MA502 ASSIGNMENTS

- I) Each student must compose his/her assignments independently. However, rough work may be done in groups;
- II) Write legibly and use only one side of each sheet of paper;
- III) Show your work. Explain your answers using FULL SENTENCES;
- IV) Late assignments will, in general, not be accepted for credit.