MA3825 Rings & Fields
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November 23, 2009

Algebra Assignment 4

DUE DATE: Monday, November 30, 2009, by 4:30pm in my office mail slot.

Please carefully review the presentation rules for homework in this course; they are reproduced on the back of this sheet for your convenience.

Please complete the following eight proofs.

1. Ex. 10 on p276.

2. Ex. 20 on p277.

3. Ex. 22 on p277.

4. Ex. 66 on p291.

5. Ex. 3 on p341.

6. Ex. 4 on p341.

7. Ex. 10 on p341.

8. Let R be the ring of upper-triangular 2×2 matrices with rational entries:

$$R = \left\{ \begin{bmatrix} a & b \\ 0 & c \end{bmatrix} \ : \ a, b, c \in \mathbb{Q} \right\} \ .$$

Let S denote the ring whose elements are the rational numbers, with ordinary addition and trivial multiplication: $ab = 0$ for all a and b in S.

Prove: *R contains an ideal A which is isomorphic to S and R/A is isomorphic to the direct sum $\mathbb{Q} \oplus \mathbb{Q}$.*

BASIC RULES FOR MA3825 ASSIGNMENTS

I) Each student must compose his/her assignments independently. However, rough work may be done in groups;

II) Write legibly and use only one side of each sheet of paper; **Any paper submitted which is sloppy or uses two sides of a page will be returned immediately with no credit.**

III) Show your work. Explain your answers using FULL SENTENCES;

IV) No late assignments will be accepted for credit.