MA3823 Group Theory
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December 4, 2012

Group Theory Assignment 5

Due Date: Wednesday December 12, 2012, by 4:30pm in my office mail slot. The faculty mailboxes are in the Department Office, SH108.

Please recall the presentation rules for assignments in this course.

1. Prove that a factor group of a cyclic group is cyclic.
2. Prove that a factor group of an Abelian group is Abelian.
3. Let G be a finite group and let H be a normal subgroup of G. Prove that the order of the element gH in G/H must divide the order of g in G.
4. Prove that the intersection of two normal subgroups is normal. (Can you generalize this?)
5. If M and N are two normal subgroups of G, then NM is also a normal subgroup of G.
6. If N is a normal subgroup of G and $|N| = 2$, prove that N is contained in the center of G.
7. Let G be a group and H an odd-order subgroup of G of index two. Show that H contains every element of G having odd order.
8. Prove that $(A \oplus B)/(A \oplus \{e\}) \cong B$.
9. Suppose there is a homomorphism from a finite group G onto Z_{10}. Prove that G has normal subgroups of indexes 2 and 5.
10. Suppose there is a homomorphism from G onto $Z_2 \oplus Z_2$. Prove that G is the union of three proper normal subgroups.