MA3823 Group Theory
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Group Theory Assignment 2

Due Date: Monday November 12, 2012, by 4:30pm in my office mail slot. The faculty mailboxes are in the Department Office, SH108.

Please recall the presentation rules for assignments in this course; they are reproduced at the end of this document for your convenience.

1. Exercise 4 on p69 (in the 8th edition).

2. **(a)** Let $(G_1, \circ)$ and $(G_2, \bullet)$ be groups. Prove that the Cartesian product $G_1 \times G_2$ forms a group under the operation $(a_1, a_2)(b_1, b_2) = (a_1 \circ b_1, a_2 \bullet b_2)$. (This new group is called the *external direct product* of G_1 and G_2 and is denoted $G_1 \oplus G_2$.)

   (b) Under what conditions is the group $G_1 \oplus G_2$ Abelian? Justify briefly.

3. Let G_1 and G_2 be groups as in Problem 2 and let H_1 be a subgroup of G_1 and let H_2 be a subgroup of G_2. Prove that $H_1 \oplus H_2$ is a subgroup of $G_1 \oplus G_2$.

4. Prove that any intersection of subgroups is a subgroup. More precisely, let G be a group and let $\{H_\alpha \mid \alpha \in I\}$ be an indexed family of subsets of G. If $H_\alpha \leq G$ for all $\alpha \in I$, then $\bigcap_{\alpha \in I} H_\alpha$ is also a subgroup of G.

5. Exercise 33 on p71.

6. Exercise 34 on p71.

7. Exercise 39 on p71.

8. Prove Theorem 3.6 in the text (p68).

9. Exercise 24 on p88.

10. Let p be a prime and let R_p denote the set of rational numbers whose denominators are relatively prime to p. Prove that R_p forms an abelian group under ordinary addtion of rational numbers. *[HINT: First, be sure to rigorously define this set.]*

PRESENTATION RULES FOR MA3823 ASSIGNMENTS

I) Each student must compose his/her assignments independently. However, rough work may be done in groups;

II) Write legibly and use only <u>one side</u> of each sheet of paper; **Any paper submitted which is sloppy or uses two sides of a page will be returned immediately with no credit.**

III) Show your work. Explain your answers using FULL SENTENCES;

IV) No late assignments will be accepted for credit.