Linear Programming
A Term, Section A01
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Week 3: Reading and Exercises

Reading

Please read Chapter 5 in the text in time for class on Monday, September 17th. But please also carefully read over the course handout entitled "Summary of the Simplex Method".

Practice Exercises

There are now many new concepts and techniques to master. Let's work on just a few of them here.

- Consider the problem

$$\begin{array}{lrcrcl} \textbf{max} & 5\,x_1 & + & x_2 & & \\ \textbf{s.t.} & 8\,x_1 & & +\,x_2 & \le & 160 \\ & 4\,x_1 & & +\,x_2 & \le & 80 \\ & x_1, & & x_2 & \ge & 0 \end{array}$$

  with optimal dictionary

```
zeta =   100.00  -   0.75 x4   -   0.25 x3
-------------------------------------------------
 x1  =    20.00  +   0.25 x4   -   0.25 x3
 x2  =     0.00  -   2.00 x4   +   1.00 x3
```

  For $\mathcal{B} = \{1, 2\}$, write down the matrix B and its inverse (without computing B^{-1} directly!).

- Consider the problem

$$\begin{array}{lrcrcl} \textbf{max} & 2\,x_1 & + & 3\,x_2 & & \\ \textbf{s.t.} & x_1 & & -2\,x_2 & \le & 9 \\ & 4\,x_1 & & +3\,x_2 & \le & 15 \\ & -\,x_1 & & +2\,x_2 & \le & 15 \\ & x_1, & & x_2 & \ge & 0 \end{array}$$

  Introduce slack variables and use linear algebra to directly write down (without pivoting!) the dictionary corresponding to basis $\mathcal{B} = \{3, 2, 5\}$. What can you say about this dictionary. *[HINT: In order to avoid the need for calculators, a few matrix inverses are computed on the second page of this handout.]*

- In the previous exercise, can you find a dictionary with basic variables x_2, x_1, x_4?

- Exercise 5.1 helps you develop an understanding of duality for real-world problems.

- Exercise 5.2.

- Apply the Complementary Slackness Conditions to decide which of the following four solutions $\mathbf{x}$ are optimal solutions for the following LP:

$$\begin{array}{lrl} \textbf{maximize} & 3x_1 \quad\; + \; x_3 & \\ \textbf{subject to} & x_1 + \; x_2 + \; x_3 & \le 12 \\ & 5x_2 - 4x_3 & \le 20 \\ & 9x_1 \quad + 2x_3 & \le 18 \\ & x_1, \;\; x_2, \;\; x_3 & \ge 0 \end{array}$$

  (a) $\mathbf{x} = [1, 7, 4]^\top$ **(b)** $\mathbf{x} = [0, 0, 9]^\top$ **(c)** $\mathbf{x} = [2, 10, 0]^\top$ **(d)** $\mathbf{x} = [0, 2, 9]^\top$

Some randomly chosen matrices and their inverses:

$$\begin{bmatrix} 1 & -2 & 1 \\ 4 & 3 & 0 \\ -1 & 2 & 0 \end{bmatrix}^{-1} = \begin{bmatrix} 0 & 2/11 & -3/11 \\ 0 & 1/11 & 4/11 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & 0 \\ 0 & 3 & 0 \\ 0 & 2 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 2/3 & 0 \\ 0 & 1/3 & 0 \\ 0 & -2/3 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -4 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$