

### Some further examples of two-player matrix games

1. An L-shaped triomino is to be placed on a  $2 \times 3$  checkerboard, covering exactly three of the six positions. Column Player secretly chooses one of the six positions. Then Row Player places the L-shaped piece in one of the eight possible ways. List all pure strategies for each player, construct the payoff matrix, and determine optimal strategies for each player. What is the value of this game?
2. Repeat the above analysis for a  $3 \times 1$  triomino placed on a  $3 \times 3$  checkerboard.
3. Row Player rolls a die and looks at the result. She then places a number of dollars on the table either equal to the roll of the die, one more than that roll, or one less. Column Player places one dollar on the table and guesses the roll of the die based on the opponent's bid. If the guess is correct, Column Player gets all the money on the table; if the guess is incorrect, Row Player gets the money on the table (i.e., wins one dollar from Column Player). How many pure strategies does each player have and what are the optimal mixed strategies for each?
4. In the game "Cheap, Middling or Dear", we saw that the analysis breaks down to looking at the three values in each group ("C", "M", "D") individually. Suppose the game is changed so that one group contains numbers 1, 6 and 7. What are the pure strategies for the players now? What are the optimal strategies and what is the value of the game?
5. Extend the game of Morra to allow each player to display one, two or three fingers. As in the text, the payoff is equal to the total number of fingers displayed (if exactly one player guesses this number). Find optimal strategies for this modified game.
6. **"Four"** A deck of four cards with values 1,2,3,4 is randomly shuffled and one card is dealt to each player. Each player can see only the card dealt to him and not the opponent's card. Row Player moves first and may either HOLD or DRAW a second card from the deck. Subsequently, Column Player faces the same choice: HOLD or DRAW. The player with the larger total wins \$1, except that a total larger than four is disqualified. Write down the pure strategies for each player and construct the full payoff matrix.