MA3231 LINEAR PROGRAMMING
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Some simple examples of two-player matrix games

1. *(One to a Hundred)* Each player secretly chooses a whole number among 1, 2, 3, . . . , 100. When the two values are revealed, the smaller one wins a dollar unless it is smaller by exactly one, in case it loses a dollar. (A tie leads to no payoff.)

2. *(Battleship)* A domino is to be placed on a 2×3 checkerboard, covering exactly two of the six positions. Column Player secretly chooses one of the six positions (the mine). Then Row Player places the domino (ship) in one of the seven possible ways. If the ship lands on the mine, then Column Player wins two points; if not, Row Player wins one point.

3. Repeat the above analysis for a 3×1 triomino placed on a 3×3 checkerboard.

4. *("Cheap, Middling or Dear")* Nine integers are written on a sheet of paper, ordered from smallest to largest, and grouped into three groups called "Cheap", "Middling" and "Dear" (expensive). Player A secretly selects one of the values (horse prices) and says "My father bought a horse at a fair". Player B asks "Cheap, Middling or Dear?" and Player A must tell B the category from which the secret number has been chosen. Player B then guesses the number. If the guess is correct, then that value determines how many points are allotted to B; if the guess is incorrect, this same number of points is allotted to A.

   Now the number just in play is struck from the list and the game is repeated for the remaining values until all nine horse prices are eliminated. (Note: This game from Argyleshire is traditionally played with numbers 1-9.)

5. *(Morra)* In this simplified version, each player displays one or two fingers and simultaneously shouts a number from two to 4. If one player correctly guesses the total and the other player does not, then that player wins a point. (How does the game change if the winnings are equal to the total number of fingers displayed?)

6. *(Four)* A deck of four cards with values 1,2,3,4 is randomly shuffled and one card is dealt to each player. Each player can see only the card dealt to him and not the opponent's card. Row Player moves first and may either HOLD or DRAW a second card from the deck. Subsequently, Column Player faces the same choice: HOLD or DRAW. The player with the larger total wins \$1, except that a total larger than four is disqualified.

7. *(A generic matrix game)* Faeze (Column Player) is to play a matrix game against a very clever opponent. The payoff matrix is as follows:

$$A = \begin{bmatrix} 2 & -2 & 4 & -1 & 2 \\ 0 & 2 & 1 & -2 & 1 \\ -3 & 4 & 0 & -2 & -2 \\ 1 & -2 & -3 & 0 & 2 \end{bmatrix}.$$

(a) Use the domination technique to reduce the above game to a 3×2 game. Briefly show your steps in words.

(b) Now find an optimal strategy for Faeze using simple optimization. (I.e., graph the constraints in the (x_1, v) plane.)

8. *(1-3 Coin)* [Here is an extremely simplified game where a random element plays a role, so the payoffs are averaged over this event. Optimization theory shows us that it is advantageous to bluff on occasion.] The numbers '1' and '3' are painted on the two sides of a fair coin. The coin is flipped and only Row Player sees the result. Row Player must then BID either \$1 or \$3. Upon seeing this bid, Column Player may either STAY IN or BOW OUT. If Column Player bows out, Column Player pays \$2 to Row Player. If Column Player stays in, then Column pays the chosen bid to Row when it matches the value on the coin and this bid is paid from Row to Column if the value on the coin does not match the bid.

   Find all strategies for both players and construct the payoff matrix (where payoffs are averaged over outcomes of the fair coin).