MA3231 Linear Programming
W. J. Martin
October 2, 2018

LP Assignment 7

DUE DATE: Tuesday, October 9th, 2018, by 5pm in my mail slot in SH108.

Please be sure to review and observe the presentation rules for assignments in this course.

Provide neat and careful solutions to Problems 1–4 (and consider Problem 5):

1.) In $\mathbb{R}^n$, find the matrix representing orthogonal projection onto the line

$$x_1 = x_2 = \cdots = x_k, \qquad x_{k+1} = x_{k+2} = \cdots = x_n = 0 \ .$$

2.) Consider the linear programming problem

$$\textbf{max} \ -x_1 - x_2 \qquad \textbf{subject to} \quad x_1 + x_2 \leq 1, \quad x_1, x_2 \geq 0.$$

(i) Starting with $x^0 = (1/2,\ 1/4)$, apply two iterations of the affine scaling method using $r = 1/2$. (First, convert to equality form.) For each iteration, give

- the constraint matrix $\hat{A}$ and objective vector $\hat{c}$ for the scaled problem;
- the projection matrix P and search direction d for this iteration;
- the ratio computation from Step 4;
- the next iterate, both in scaled form $\hat{x}$ and as a solution x^{k+1} to the original problem above. (So you will be finding x^1 and x^2.)

(ii) On a sheet of graph paper, make a careful (and large!) drawing of the feasible region in $\mathbb{R}^2$. For x^0 and each of the next two iterates, x^1, and x^2, plot both the gradient of the objective function (namely $c^\top = [-1\ \ -1]$) at that point as well as the scaled step direction $x^{k+1} - x^k$.

3.) Repeat the steps of Problem 2 (again with $r = 1/2$) for the linear programming problem

$$\begin{array}{lrcrcrcr}
\textbf{max} & x_1 & + & x_2 & + & x_3 & & \\
\textbf{s.t.} & x_1 & & & + & x_3 & = & 3 \\
 & -2\,x_1 & + & x_2 & - & x_3 & = & -4 \\
 & x_1, & & x_2, & & x_3 & \geq & 0
\end{array}$$

starting with $x^0 = (2, 1, 1)$ (a column vector). (You may wish to use MAPLE to check your computations and to follow the trajectory further.)

4.) Perform two iterations of the affine scaling method for the problem

$$\begin{array}{rrcrl}
\textbf{maximize} & 2x_1 + 2x_2 - & x_3 & & \\
\textbf{subject to} & x_1 & & & = 5 \\
 & & x_2 + 2x_3 & & = 7 \\
 & x_1, \quad x_2, & x_3 & & \geq 0
\end{array}$$

with step size $r = 1/2$ and initial solution $x^0 = (5, 3, 2)$. Show your work as in Problem 2.

5.) [**BONUS QUESTION**] Exercise 21.2 in the text.