MA3231 Linear Programming
W. J. Martin
September 27, 2018

LP Assignment 6

DUE DATE: Thursday October 4th at the beginning of class.

Please carefully review the assignment presentation rules for this course.

Provide neat and careful solutions to the following five problems:

1.) Consider the matrix game with payoff matrix

$$A = \begin{bmatrix} 3 & -1 \\ -2 & 2 \\ -3 & 4 \end{bmatrix}$$

Set up the LP formulation for Row Player's optimal strategy. Solve by the simplex method, showing each dictionary and each pivot.

2.) Consider the matrix game with payoff matrix

$$A = \begin{bmatrix} 4 & 0 & 4 & 8 & -3 \\ 0 & -2 & 4 & 6 & -4 \\ -6 & 2 & -6 & -8 & 1 \\ -4 & 3 & -4 & -4 & 1 \\ 0 & -4 & -3 & 3 & -4 \end{bmatrix}$$

First apply the domination technique to reduce the game (show your steps). Then find the value of the game and optimal strategies for each player. You may check your answer using a computer, but please apply the simplex method to reach optimality.

3.) *(One, Five or Ten)* Each player puts a bill on the table, either a \$1 bill, a \$5 bill, or a \$10 bill. If the total number of dollars on the table is odd, then Row Player collects the total; if even, Column Player takes the pot. (For example, if Row puts her \$5 on the table and Column puts his \$10 on the table, the sum is odd; so Row gets the \$10 and the payoff is -10.)
(a) Column Player has the task of finding an optimal strategy. Formulate this as a linear programming problem.
(b) Apply the domination technique to reduce this to a 2×2 game.
(c) Apply the Simplex Method (by hand or computer, but show the dictionaries) to find an optimal strategy.

(d) Explain, in words, the optimal strategy for each player. Is this game fair? Is it symmetric?

4.) Find optimal strategies for Rock/Paper/Scissors where the payoff is one dollar if Rock or Scissors wins and m dollars if Paper wins, where m is a free parameter. (Note: the rules as to who wins are the same as in the traditional game. Only the amount to be paid is changed.) Is it still a fair game? *[HINT: Use MAPLE to find optimal strategies for various fixed values of m and then make a conjecture for general m. Verify your conjecture using the Minimax Theorem.]*

5.) Playing on a 2×3 checkerboard, Row Player secretly chooses a square and writes it down. Column Player then places a domino on the board, exactly covering two adjacent squares. If the secret square gets covered, then Row Player wins three dollars; if the ship (domino) avoids the mine (secret square), then Column Player wins one dollar. Find the value of the game and optimal strategies for both players. Finally, informally extend your analysis to bigger checkerboards.