

LP Assignment 2

DUE DATE: Tuesday, September 4th, 2018, at the beginning of class.

Please **carefully** read the presentation rules at the bottom of the second page.

Please complete the following five problems.

1. (a) Apply the simplex method to solve the following LP:

$$\begin{aligned} & \mathbf{maximize} && 3x_1 - x_2 \\ & \mathbf{subject \ to} && -x_1 + x_2 \geq -8 \\ & && -2x_1 + x_2 \geq -20 \\ & && x_2 \leq 8 \\ & && x_1, x_2 \geq 0 \end{aligned}$$

HINT: Multiply both sides of the first two constraints by -1 to convert this to standard form before introducing your slack variables.

(b) Carefully plot the feasible region on a separate sheet of paper and mark off the location of each basic feasible solution corresponding to the dictionaries in your computation above.

2. Apply the simplex method to solve the following LP:

$$\begin{aligned} & \mathbf{maximize} && 100x_1 + 90x_2 \\ & \mathbf{subject \ to} && 10x_1 + 8x_2 \leq 360 \\ & && 4x_1 + 3x_2 \leq 132 \\ & && x_1, x_2 \geq 0 \end{aligned}$$

3. First convert to standard form and then apply the simplex method:

$$\begin{aligned}
\mathbf{maximize} \quad & -x_1 + 2x_2 + 3x_3 \\
\mathbf{subject \ to} \quad & -2 + x_2 + x_3 \leq x_1 \\
& -6 + x_1 + x_3 \leq x_2 \\
& -8 + x_1 + x_2 \leq x_3 \\
& x_1, x_2, x_3 \geq 0
\end{aligned}$$

4. Apply the two-phase simplex method to find an optimal solution to the following LP:

$$\begin{aligned}
\mathbf{max} \quad & 5x_1 + 2x_2 \\
\mathbf{s.t.} \quad & 2x_1 + x_2 \leq 100 \\
& -2x_1 - x_2 \leq -50 \\
& x_1, x_2 \geq 0
\end{aligned}$$

5. Your company uses three resources — snot, spit and filth — to manufacture three products — goodness, compassion and cheer. Let

- x_1 denote the number of expressions of goodness to be produced;
- x_2 denote the number of acts of compassion to be produced;
- x_3 denote the number of bundles of cheer to be produced.

With 120 and 240 gallons, respectively, of snot and spit and and 180 tons of filth available, the following LP formulation uses per-unit profit for each product in order to maximize overall profit (in euros) subject to resource constraints:

$$\begin{aligned}
\mathbf{maximize} \quad & 20x_1 + 30x_2 + 40x_3 \\
\mathbf{subject \ to} \quad & x_1 + x_2 + x_3 \leq 120 \\
& 2x_1 - x_2 + 3x_3 \leq 240 \\
& 2x_1 + 4x_2 \leq 180 \\
& x_1, x_2, x_3 \geq 0
\end{aligned}$$

(For example, each expression of goodness is ingeniously made from one gallon of snot, two gallons of spit and two tons of filth.)

Apply the simplex method to find the optimal production strategy. Be sure to give the dictionaries for each pivot. Along with the optimal dictionary, give the optimal solution and profit in words (e.g., produce 40 acts of compassion, etc., using a total of 120 gallons of spit, etc., for a total profit of 3000 euros).

You may use software to check your work on this problem, but please develop the skill to pivot by hand.

BASIC RULES FOR LP ASSIGNMENTS

- I)** Each student must compose his/her assignments independently. However, rough work may be done in groups;
- II)** Write legibly and use only one side of each sheet of paper;
- III)** Show your work. Explain your answers using FULL SENTENCES;
- IV)** Late assignments will, in general, not be accepted for credit.