

MA2210 Assignment 3

DUE DATE: Thursday, March 29, 4:00pm.

Please recall the presentation rules for the course.

Please complete the following four problems (since the first one is long, it will count as two problems in grading):

1. Your company uses three resources — snot, spit and filth — to manufacture three products — goodness, compassion and cheer. The goal is to maximize profit (also called self-worth) subject to resource constraints. Let
 - x_1 denote the number of units (“worlds”) of goodness to be produced;
 - x_2 denote the number of units (“lives”) of compassion to be produced;
 - x_3 denote the number of units (“bundles”) of cheer to be produced.

The following LP formulation describes your company’s overall profit from these products and their use of resources, which are all available only in limited quantities (measured in gallons):

$$\begin{array}{ll} \textbf{maximize} & 20x_1 + 30x_2 + 40x_3 \\ \textbf{subject to} & x_1 + x_2 + x_3 \leq 120 \\ & 2x_1 - x_2 + 3x_3 \leq 240 \\ & 2x_1 + 4x_2 \leq 180 \\ & x_1, x_2, x_3 \geq 0 \end{array}$$

For example, each bundle of cheer uses one gallon of snot, 3 gallons of spit and no filth whatsoever, providing \$40 in profit.

(a) Introduce slack variables and enter the problem into a computer. Using software, find the optimal tableau for this problem.

(b) Suppose that an extra gallon of snot becomes available to you. How much would you be willing to pay for it and why? Up to what limit is this price sensible? Do the same for spit and filth.

(c) Suppose that the (per world) profit for goodness changes from \$20 to \$35. Would you start producing goodness or not? Explain.

(d) Over what range of per-unit profits c_2 is the given tableau still optimal? Explain. Do the same for c_3 .

(e) Suppose that, after optimizing, you learn that the profit per unit for cheer has diminished from 40 to 10 dollars per world. Write down the modified final tableau. If it is no longer optimal, then pivot to optimality.

Extra credit: (Just for those interested) Suppose that, after solving the original LP, the amount of spit available is reduced from 240 gallons to 160 gallons. Work out the modified final tableau. Apply the dual simplex method¹ to pivot from this tableau to optimality. What is the smallest number of gallons b_2 of spit for which the basis $\mathcal{B} = \{2, 3, 6\}$ is optimal?

2. An iron foundry has a firm order to produce 1000 pounds of castings containing at least 0.45 percent manganese and between 3.25 percent and 5.50 percent silicon. As these particular castings are special order, there are no suitable castings on hand. The castings sell for \$0.90 per pound. The foundry has three types of pig iron available in essentially unlimited amounts, with the following properties:

	<i>Type of pig iron</i>		
	A	B	C
Silicon	4 %	1 %	0.6%
Manganese	0.45%	0.5%	0.4%

Further, the production process is such that pure manganese can also be added directly to the melt. The costs of the various possible inputs are:

Pig A	\$48 per thousand lbs.
Pig B	\$60 per thousand lbs.
Pig C	\$30 per thousand lbs.
Manganese	\$8 per lb.

It costs one cent to melt down a pound of pig iron. Out of what inputs should the foundry produce the castings in order to maximize profits?

Set this up as a linear programming problem. Be sure to define your variables. Then solve using software and include a printout of the Answer Report or final tableau.

3. Consider the LP problem in Problem 2 above, where three types of pig iron are to be blended, with the possible addition of pure manganese to build 1000 lbs of castings.

¹The dual simplex method pivots on the dual LP using the information as presented in the primal tableau. As you might guess, this method assumes all $\tilde{c}_j \leq 0$ and pivots until all $\tilde{b}_i \geq 0$. The leaving variable is chosen first, as one i with a negative right-hand side. Then ratios are computed to find an entering variable so as to keep all reduced costs non-positive. Pivoting is done as usual.

(a) Describe the optimal solution in sentence form. List each constraint in a table and the corresponding value of its slack variable. In the same table, give the per-unit cost of each of these restrictions to your operation.

(b) The cost of Pig Iron B is currently 6 cents a pound. The supplier wishes to know at what price it will become profitable for you to use Pig B in your blend. And how much of it will you purchase at this price? (Perform only one pivot.)

(c) The customer is unhappy with your \$900 price quote for the castings. She offers to weaken the upper limit on silicon concentration from 5.5% to 6.5% if you'll sell the castings for \$850. What is your response?

4. Exercise #3 on p346 in the text.