Linear Algebra
C Term, Sections C01-C04
W. J. Martin
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Linear Algebra Quiz 5

Sample Solutions

Use elementary row operations to compute the determinant $\det A$ of the following matrix:

$$A = \begin{bmatrix} 2 & -1 & 3 & 0 & 6 & 1 \\ 2 & -1 & 3 & 0 & 6 & 6 \\ 4 & -2 & 5 & 1 & 13 & 3 \\ -2 & 1 & -3 & 1 & -6 & -1 \\ -6 & 3 & -9 & 0 & -13 & 2 \\ 2 & 1 & 5 & 2 & 8 & 3 \end{bmatrix}$$

Solution: We make up a name for the determinant of A, call it Δ, and we keep track of the determinant as we row reduce to upper-triangular form:

$$A = \begin{bmatrix} 2 & -1 & 3 & 0 & 6 & 1 \\ 2 & -1 & 3 & 0 & 6 & 6 \\ 4 & -2 & 5 & 1 & 13 & 3 \\ -2 & 1 & -3 & 1 & -6 & -1 \\ -6 & 3 & -9 & 0 & -13 & 2 \\ 2 & 1 & 5 & 2 & 8 & 3 \end{bmatrix} \qquad \det = \Delta$$

$$\sim \begin{bmatrix} 2 & -1 & 3 & 0 & 6 & 1 \\ 0 & 0 & 0 & 0 & 0 & 5 \\ 0 & 0 & -1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 & 5 \\ 0 & 2 & 2 & 2 & 2 & 2 \end{bmatrix} \begin{matrix} (R1) \\ (R2)-(R1) \\ (R3)-2(R1) \\ (R4)+(R1) \\ (R5)+3(R1) \\ (R6)-(R1) \end{matrix} \qquad \det = \Delta \quad \text{(no change)}$$

$$\sim \begin{bmatrix} 2 & -1 & 3 & 0 & 6 & 1 \\ 0 & 2 & 2 & 2 & 2 & 2 \\ 0 & 0 & -1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 & 5 \\ 0 & 0 & 0 & 0 & 0 & 5 \end{bmatrix} \begin{matrix} \\ (R6) \\ \\ \\ \\ (R2) \end{matrix} \qquad \det = -\Delta \quad \text{(row swap)}$$

Now we have arrived at an upper-triangular matrix. We know that its determinant is just the product of its diagonal entries. So this last matrix has determinant

$$\begin{aligned} 2 \cdot 2 \cdot (-1) \cdot 1 \cdot 5 \cdot 5 &= -100 \\ -\Delta &= -100 \\ \Delta &= 100 \end{aligned}$$

So we have found $\det A = 100$.