Linear Algebra
C Term, Sections C01-C04
W. J. Martin
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Linear Algebra Quiz 4

Sample Solutions

1.) Compute B^{-1} and C^{-1} for the following matrices. Show your process, but you do not need to annotate your individual row operations.

Solution:

$$[B|I] = \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & -1 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & 3 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1 & 0 & 2 \\ 0 & 1 & 0 & 2 & 1 & -3 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

$$B^{-1} = \begin{bmatrix} -1 & 0 & 2 \\ 2 & 1 & -3 \\ -1 & 0 & 1 \end{bmatrix}$$

$$[C|I] = \left[\begin{array}{cccc|cccc} 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -4 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 3 & 3 & 0 & 0 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{cccc|cccc} 1 & 0 & 3 & 3 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & -4 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 \end{array}\right] \begin{array}{c} (R4) \\ (R2) \\ (R1) \\ (R3) \end{array}$$

$$\sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -3 & 0 & 0 & 1 \\ 0 & 1 & 0 & -4 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -3 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & -4 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \end{array}\right]$$

$$C^{-1} = \begin{bmatrix} -3 & 0 & 0 & 1 \\ 0 & 1 & -4 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

2.) If

$$A = \begin{bmatrix} 1 & -3 & 0 & 1 & 1 & 4 \\ 1 & 0 & 1 & -1 & -1 & 2 \\ 0 & 1 & -2 & 4 & 3 & 1 \end{bmatrix}, \qquad MA = \begin{bmatrix} 0 & -7 & 0 & 0 & 1 & 3 \\ 1 & 4 & 0 & 1 & 0 & 1 \\ 0 & -11 & 1 & -2 & 0 & 4 \end{bmatrix}$$

then find the matrix M.

Solution: We look at columns 5, 1 and 3 (in that order) of the product MA to see that

$$M \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix} = \mathbf{e}_1, \qquad M \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \mathbf{e}_2, \qquad M \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix} = \mathbf{e}_3.$$

Piecing this information together, we see that

$$M\begin{bmatrix} 1 & 1 & 0 \\ -1 & 1 & 1 \\ 3 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

so M is just the inverse of this 3×3 matrix (<u>if</u> M exists). We just invert

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ -1 & 1 & 1 & 0 & 1 & 0 \\ 3 & 0 & -2 & 0 & 0 & 1 \end{array}\right] = \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 2 & 1 & 1 & 1 & 0 \\ 0 & -3 & -2 & -3 & 0 & 1 \end{array}\right] = \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 2 & 1 & 1 & 1 & 0 \\ 0 & -3 & -2 & -3 & 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & 0 \\ 0 & 1 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & -\frac{1}{2} & -\frac{3}{2} & \frac{3}{2} & 1 \end{array}\right] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & -2 & -1 \\ 0 & 1 & 0 & -1 & 2 & 1 \\ 0 & 0 & 1 & 3 & -3 & -2 \end{array}\right]$$

and verify that

$$MA = \begin{bmatrix} 2 & -2 & -1 \\ -1 & 2 & 1 \\ 3 & -3 & -2 \end{bmatrix}\begin{bmatrix} 1 & -3 & 0 & 1 & 1 & 4 \\ 1 & 0 & 1 & -1 & -1 & 2 \\ 0 & 1 & -2 & 4 & 3 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -7 & 0 & 0 & 1 & 3 \\ 1 & 4 & 0 & 1 & 0 & 1 \\ 0 & -11 & 1 & -2 & 0 & 4 \end{bmatrix},$$

which it does.