Linear Algebra
C Term, Sections C01-C04
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Linear Algebra Quiz 1
Sample Solutions

Consider the following system of linear equations:

$$\begin{array}{rrrcr} x_1 & +2x_2 & -2x_3 & = & 9 \\ 3x_1 & +6x_2 & + x_3 & = & 34 \\ -2x_1 & -4x_2 & +5x_3 & = & -17 \end{array}$$

1.) Write down the augmented matrix corresponding to this system.

Solution: We put the coefficients of the x_j on the left and the right-hand side constants on the right, separated by a vertical line (which, in the text, is dashed):

$$[A|b] = \left[\begin{array}{rrr|r} 1 & 2 & -2 & 9 \\ 3 & 6 & 1 & 34 \\ -2 & -4 & 5 & -17 \end{array}\right]$$

2.) Perform Gauss-Jordan elimination on this matrix to obtain a matrix in **reduced row echelon form**.

Solution:

$$[A|b] = \left[\begin{array}{rrr|r} 1 & 2 & -2 & 9 \\ 3 & 6 & 1 & 34 \\ -2 & -4 & 5 & -17 \end{array}\right] \sim \left[\begin{array}{rrr|r} 1 & 2 & -2 & 9 \\ 0 & 0 & 7 & 7 \\ 0 & 0 & 1 & 1 \end{array}\right] \begin{array}{l} \\ (R2) - 3(R1) \\ (R3) + 2(R1) \end{array}$$

$$\sim \left[\begin{array}{rrr|r} 1 & 2 & -2 & 9 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} \\ \frac{1}{7}(R2) \\ (R3) - \frac{1}{7}(R2) \end{array} \sim \left[\begin{array}{rrr|r} 1 & 2 & 0 & 11 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] (R1) + 2(R2)$$

We have reached RREF.

3.) Using the result of (2) above, find all solutions to the original linear system.

Solution: Now we easily read off the solutions. Columns 1 and 3 are pivot columns; x_2 is the only free variable. So there are infinitely many solutions, given by

$x_1 = 11 - 2r$
$x_2 = r$ (free)
$x_3 = 1$
where r is any real number.