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How to Multiply Matrices

First, some simple terminology.

- We said that an $m \times n$ matrix is a rectangular array of real numbers arranged in m rows and n columns
- the "(i, j)-entry" of a matrix refers to the number in row i, column j.

So if A and B are matrices (see Note), then the (i, j)-entry of their product AB is just the dot product (as defined in class) of row i of A with column j of B.

Example: If A and B are the 2×2 matrices

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \qquad B = \begin{bmatrix} 5 & 0 \\ 1 & 6 \end{bmatrix},$$

then we compute the product

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & 0 \\ 1 & 6 \end{bmatrix}$$

entry by entry:

$(1, 1)$-entry $= (1, 2) \cdot (5, 1)$ (dot product of row 1 of A with col 1 of B)
$(AB)_{11} = 7$

$(1, 2)$-entry $= (1, 2) \cdot (0, 6)$
$(AB)_{12} = 12$

$(2, 1)$-entry $= (3, 4) \cdot (5, 1)$
$(AB)_{12} = 19$

$(2, 2)$-entry $= (3, 4) \cdot (0, 6)$
$(AB)_{12} = 24$

So, putting this all together, we have our first matrix multiplication:

$$AB = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & 0 \\ 1 & 6 \end{bmatrix} = \begin{bmatrix} 7 & 12 \\ 19 & 24 \end{bmatrix}.$$

Note: If the number of columns of A is not the same as the number of rows of B, these dot products don't make sense. So the matrix product is undefined. In other words, AB is only defined when the number of columns of A equals the number of rows of B.