Matrices & Linear Algebra I
C Term, Sections C01-C04
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February 22, 2010

Week 6: Reading and Exercises

Reading

Recall that you were previously instructed to read Sections 4.1 to 4.7. We have only covered a few of these topics so far, so I think we will get to Section 5.1 this week, but certainly no further. So please read Sections 4.3–4.7 again and read Section 5.1 very carefully.

We have already introduced the main concepts in Chapter 5, so we will be able to cover Sections 5.2–5.4 next week before the third test. It may be advisable to give those sections a look as well, over the weekend.

The capstone portion of the course is a basic introduction to eigenvalues and eigenvectors. A non-zero vector $\mathbf{v}$ in $\mathbb{R}^n$ is an *eigenvector* for an $n \times n$ matrix A if there is a scalar θ for which

$$A\mathbf{v} = \theta\mathbf{v}.$$

In this case, θ is called an *eigenvalue* of A. If we view A as a geometric transformation mapping $\mathbb{R}^n$ to itself, then the eigenvectors are "stable" vectors: they don't get twisted at all by A, but are simply scaled by a constant. For example, if A effects a rotation in $\mathbb{R}^3$ about the origin, then any vector along the axis of rotation is an eigenvector with eigenvalue one.

We can also define eigenvalues and eigenvectors for any linear transformation T from a vector space to itself, $T : V \to V$. A non-zero vector $\mathbf{v}$ is an eigenvector for T if $T(\mathbf{v}) = \lambda\mathbf{v}$ for some scalar λ, which is the corresponding eigenvalue. I give an interesting example below.

Practice Exercises

NOTE: Do not hand in.

If you have any questions about these problems, please discuss them in your conference or come see Erin or me.

- pages 308-9, #1-23 (odd), 21, 32.
- pages 317-8, #1-17 (odd).
- pages 325-6, #1-31 (odd).
- pages 333-4, #1-13 (odd).

Here's an important example. We can consider the vector space V of "smooth" functions[1] and view each function $\mathbf{v} = e^{\lambda x}$ as an eigenvector for the linear transformation

$$T : V \to V \qquad \text{via} \qquad T\left(f(x)\right) = \frac{df}{dx}$$

which takes a function to its derivative: indeed, $\frac{d}{dx}(e^{\lambda x}) = \lambda e^{\lambda x}$ is written in vector space notation as $T(\mathbf{v}) = \lambda\mathbf{v}$.

[1] Please ignore this technical detail. In order for the derivative to map V to itself, we need to restrict V to the set of functions $f : \mathbb{R} \to \mathbb{R}$ which are not only differentiable, but also have second derivatives, third derivatives, and so on, in the same set V. This space is called C^∞ for those who are curious.