Linear Algebra
C Term, Sections C01-C04
W. J. Martin
February 28, 2010

Examples: Computing Eigenvalues

I give twelve matrices. For each matrix, find **all** the eigenvalues and as many linearly independent eigenvectors as you can. (In most cases given here, an $n \times n$ matrix has n linearly independent eigenvectors.)

NOTE: The solutions will be given in another document. But you should of course attempt to solve a problem for yourself before you look at the answer.

The 2×2 case

Let's start with 2×2 matrices. Recall that the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ has determinant $ad - bc$.

Example 1: Find all the eigenvalues of $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

Example 2: Find all the eigenvalues of $A = \begin{bmatrix} 3 & 2 \\ 8 & 3 \end{bmatrix}$

Example 3: Now let's try $A = \begin{bmatrix} -1 & 4 \\ 4 & 5 \end{bmatrix}$

Example 4: Here's an easy one: $A = \begin{bmatrix} -15 & 0 \\ 0 & -15 \end{bmatrix}$

Example 5: This one looks easy at first: $A = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$. But what about its eigenvectors?

Example 6: This one is a trick. Find all the eigenvalues of $A = \begin{bmatrix} 5 & -2 \\ 3 & 5 \end{bmatrix}$

The 3×3 case

Example 7: Find all the eigenvalues of $A = \begin{bmatrix} 1 & 3 & 2 \\ 1 & 3 & 2 \\ 0 & 5 & 1 \end{bmatrix}$

Example 8: Find all the eigenvalues of $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix}$

Example 9: Find all the eigenvalues of $A = \begin{bmatrix} 3 & 0 & 7 \\ 1 & 8 & -6 \\ 1 & 0 & 9 \end{bmatrix}$ *[HINT: What is special about the second column?]*

Upper-triangular matrices

Example 10: Find all the eigenvalues of

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 3 & 4 \\ 0 & 0 & 3 & 4 \\ 0 & 0 & 0 & 4 \end{bmatrix}$$

Example 11: Find all the eigenvalues of

$$A = \begin{bmatrix} 1 & 2 & -1 & 0 & 1 & 0 \\ 0 & 5 & 5 & 1 & 2 & 3 \\ 0 & 0 & 1 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 & 4 & 4 \\ 0 & 0 & 0 & 0 & 4 & 2 \\ 0 & 0 & 0 & 0 & 0 & 5 \end{bmatrix}$$

Example 12: Find all the eigenvalues of

$$A = \begin{bmatrix} 3 & 1 & 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 1 & 0 & 0 \\ 0 & 0 & 0 & 5 & 1 & 0 \\ 0 & 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 7 \end{bmatrix}$$