Linear Algebra
C Term, Sections C01-C04
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Linear Algebra Assignment 7

DUE DATE: Tuesday, March 2, 4:30pm. Place in mail bin labelled "MA2071" in Room SH108. There, you will find a separate folder for each section. (Be sure to write your name and your section number on the 1st page.)

Please recall Professor Martin's rules for Linear Algebra assignments.

Please complete the following five problems.

1. Consider the vector space $\mathbb{P}_3$ and the two ordered bases

$$\mathcal{B} = \left\{1,\ t+1,\ t^2+2t+1,\ t^3+3t^2+3t+1\right\}$$

and

$$\mathcal{C} = \left\{t^3,\ t^3-1,\ t^3-t,\ t^3-t^2\right\}$$

for this space.

For each of the following vectors, find the *coordinate vector* of it relative to each of the bases $\mathcal{B}$ and $\mathcal{C}$. Show your work.

(a) $\mathbf{u} = 3 + 3t + t^2$ (find $[\mathbf{u}]_{\mathcal{B}}$ and $[\mathbf{u}]_{\mathcal{C}}$)
(b) $\mathbf{v} = t^3$ (find $[\mathbf{v}]_{\mathcal{B}}$ and $[\mathbf{v}]_{\mathcal{C}}$)
(c) $\mathbf{w} = 1 + t + t^2$ (find $[\mathbf{w}]_{\mathcal{B}}$ and $[\mathbf{w}]_{\mathcal{C}}$)

2. **(a)** Derive the change-of-coordinates matrix from $\mathcal{B}$ to $\mathcal{C}$, where $\mathcal{B}$ and $\mathcal{C}$ are the bases given in Problem 1.

(b) Using part (a), derive the change-of-coordinates matrix from $\mathcal{C}$ to $\mathcal{B}$.

3. For each of the following matrices, compute the characteristic equation of the given matrix and use this to find all eigenvalues of the matrix (including multiplicities).

   (a) $A = \begin{bmatrix} 1 & -5 \\ -2 & 4 \end{bmatrix}$

   (b) $B = \begin{bmatrix} -1 & -1 \\ 1 & -3 \end{bmatrix}$

   (c) $C = \begin{bmatrix} 2 & 0 & -1 & 3 \\ 0 & 3 & 1 & 1 \\ 0 & 0 & 4 & 6 \\ 0 & 0 & 0 & 4 \end{bmatrix}$

   (d) $D = \begin{bmatrix} 1 & 0 & -2 \\ 5 & 0 & 0 \\ -3 & 0 & 2 \end{bmatrix}$

4. For each of the following matrices, find a basis for the eigenspace V_λ where $\lambda = 5$. (You are assured, without any need for computation, that $\lambda = 5$ is indeed an eigenvalue.)

   (a) $A = \begin{bmatrix} 6 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$

   (b) $B = \begin{bmatrix} 6 & 1 & 0 & 0 \\ 0 & 6 & 1 & 0 \\ 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 5 \end{bmatrix}$

   (c) $C = \begin{bmatrix} 5 & 54 & -8 & -46 \\ 0 & -19 & 4 & 20 \\ 0 & -30 & 5 & 30 \\ 0 & -18 & 4 & 19 \end{bmatrix}$

5. Diagonalize the following matrix:

$$A = \begin{bmatrix} 17 & -12 & 36 \\ 24 & -19 & 48 \\ 2 & -2 & 3 \end{bmatrix}$$

   (That is, find a diagonal matrix D and an invertible matrix P such that $A = PDP^{-1}$.)