Linear Algebra
C Term, Sections C01-C04
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Linear Algebra Assignment 5

Due Date: Tuesday, February 16, 4:30pm. Place in mail bin labelled "MA2071" in Room SH108. There, you will find a separate folder for each section. PLEASE WRITE YOUR NAME AND SECTION NUMBER ON YOUR ASSIGNMENT.

Please recall Professor Martin's rules for Linear Algebra assignments.

Please complete the following five problems.

1. Find the 3×3 matrix that produces the following composite 2D transformation by acting on homogeneous coordinates:

   Translate by $(2, -5)$ *and then reflect about the line* $x = -3$.

   Be sure to show your work. You should give matrices for each of the two individual transformations and then give the matrix for the composite function.

2. Find the 4×4 matrix that produces the following 3D transformation using homogeneous coordinates:

   Rotate by $30°$ *about the line joining* $(0, 1, 0)$ *to* $(2, 1, 0)$*, in a clockwise direction when viewed from an observer located at* $(2, 1, 0)$ *looking at the plane* $x = 0$.

   Again, show your work. In this case, a rotation matrix is conjugated by a translation.

3. # 12 on p166. (Cf. #11, the solution of which is outlined in the back of your book. For #12, you'll need a half-angle formula for $\tan \theta$.)

4. #18 on p166

5. This problem has four parts, (a)–(d). For each of the following structures $(V, \oplus, \odot)$, show that it is **not** a vector space by identifying a specific axiom that is violated. Give specific vectors in your counterexample.

(a) $V = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} \;\middle|\; a, b, c \in \mathbb{R} \right\}$ together with the operations

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} \oplus \begin{bmatrix} d \\ e \\ f \end{bmatrix} = \begin{bmatrix} a+d \\ b+e \\ 0 \end{bmatrix} \qquad \text{and} \qquad r \odot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} ra \\ rb \\ rc \end{bmatrix}$$

(b) $V = \left\{ \begin{bmatrix} a \\ b \end{bmatrix} \;\middle|\; a, b > 0 \right\}$ together with the operations

$$\begin{bmatrix} a \\ b \end{bmatrix} \oplus \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} ac \\ bd \end{bmatrix} \qquad \text{and} \qquad r \odot \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} ra \\ rb \end{bmatrix}.$$

(c) $V = \left\{ \begin{bmatrix} a \\ b \end{bmatrix} \;\middle|\; a, b \neq 0 \right\}$ together with the operations

$$\begin{bmatrix} a \\ b \end{bmatrix} \oplus \begin{bmatrix} c \\ d \end{bmatrix} = \begin{bmatrix} ac \\ bd \end{bmatrix} \qquad \text{and} \qquad r \odot \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} ra \\ rb \end{bmatrix}$$

for $r \neq 0$ and $0 \odot \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

(d)

$$V = \{T : \mathbb{R}^2 \to \mathbb{R}^3 \mid T \text{ is a linear trans.}\}$$

together with the operations

$$(T \oplus T')(\mathbf{x}) = T(\mathbf{x}) + T'(\mathbf{x})$$

for $\mathbf{x}$ in $\mathbb{R}^2$ and

$$(r \odot T)\mathbf{x} = T\left(\begin{bmatrix} rx_2 \\ rx_1 \end{bmatrix} \right).$$