Linear Algebra
C Term, Sections C01-C04
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January 14, 2010

Linear Algebra Assignment 1

DUE DATE: Wednesday, January 20, 4:30pm. Place in mail bin labelled "MA2071" in Room SH108. (The label is red and the bin is near the bottom row, underneath the staff mail slots.)

PROF. MARTIN'S RULES FOR LINEAR ALGEBRA ASSIGNMENTS:

I) Each student must compose his/her assignments independently. However, brainstorming may be done in groups;

II) **Use only one side of each sheet of paper**. Write legibly using correct English;

III) Show your work. Explain your answers using FULL SENTENCES;

IV) Use a staple when you submit more than one sheet and want them all back. There is a stapler for public use in the Mathematical Sciences Department Office (SH108) where assignments are turned in;

V) **No late assignments will be accepted for credit.**

Please complete the following five problems. (Start thinking about them right away; please don't wait until the last day.)

1. Consider the following system of linear equations:

$$\begin{array}{rrrrrrr} 2x_1 & -2x_2 & +2x_3 & & -2x_5 & = & 16 \\ x_1 & +\,x_2 & +5x_3 & & +9x_5 & = & 8 \\ -x_1 & & -3x_3 & +x_4 & +2x_5 & = & -1 \\ x_1 & & +3x_3 & & +4x_5 & = & 8 \end{array}$$

(a) Write down the augmented matrix corresponding to this system.

(b) Use the row reduction algorithm on this matrix to obtain a row equivalent matrix in **reduced row echelon form**.

(c) Using part (b), find all solutions to the original linear system. Describe the solution set in parametric form as in (5) and (7) on p21-22 in the text. (I encourage you to use letters such as r, s and t for parameters rather than x_i as the book does.)

2. Consider the linear system with augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 1 & 5 & k \\ 2 & h & 7 & 10 \\ 3 & 3 & 15 & 12 \end{array}\right].$$

(a) For which values of h and k is the system inconsistent? Explain.

(b) For which values of h and k does the system have a unique solution? Explain.

(c) For which values of h and k does the system have infinitely many solutions? Explain.

3. Exercise 12 on p64. (The system of equations that you must solve is simply derived from the principle that, at each of the four network nodes A, B, C, D, the flow into that node (in cars/min.) must equal the flow out of that node. For example, at node B, we obtain the equation $x_1 + x_2 = 200$. As always, show your work.

4. Not all algebraic systems are as intuitive as the real numbers. For example, if $A = \begin{bmatrix} 6 & 9 \\ -4 & -6 \end{bmatrix}$, then $A^2 := AA = 0$ where 0 denotes the 2×2 matrix of all zeros. In fact, there are infinitely many 2×2 matrices A satisfying $A^2 = 0$. Systematically determine all 2×2 matrices $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ which satisfy $A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$. [HINT: Split your analysis into two case: one where $b = 0$ and in the second case assume $b \neq 0$.]

5. *In this course, we will discuss vector spaces other than the obvious one consisting of all $n \times 1$ matrices. For example, the set of all continuous functions defined on a given real interval is a vector space.*

   In this problem. we will analyze the following "dot product" on the space of all continuous functions on $[0, 1]$:

$$\langle f, g \rangle := \int_0^1 f(x)g(x)\,dx.$$

   Two vectors in a vector space are **orthogonal** if their dot product is zero. (E.g., $cos(\pi x)$ is orthogonal to $\sin(\pi x)$ in this vector space.) By computing the above dot product for each of the six possible pairs[1], determine which of the following functions are orthogonal to eachother:

[1] That is, simply compute the integrals defined by $\langle a, b \rangle$, $\langle a, c \rangle$, $\langle a, d \rangle$, $\langle b, c \rangle$, $\langle b, d \rangle$ and $\langle c, d \rangle$

(a) $a(x) = 1$ for $0 \leq x \leq 1$

(b) $b(x) = x$ for $0 \leq x \leq 1$

(c) $c(x) = x - \frac{1}{2}$ for $0 \leq x \leq 1$

(d) $d(x) = 6x^2 - 6x + 1$ for $0 \leq x \leq 1$

NOTE: In signal processing, signals represented by orthogonal functions can be easily separated by a filter. This is how a cell phone network can allow multiple users on the same frequency.