Linear Algebra
C Term, Sections C01-C04
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Sample Solutions – Quiz 6

We are given the matrix

$$A = \begin{bmatrix} 2 & -4 & -1 & 3 & 11 & 2 \\ -1 & 2 & 0 & 4 & 11 & 4 \\ 3 & -6 & -6 & -6 & -15 & -12 \\ -2 & 4 & 1 & 1 & 1 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

(a) Find a basis for the row space of A

SOLUTION: We take the non-zero rows of the reduced row echelon form:

$$S = \{ \begin{bmatrix} 1 & -2 & 0 & 0 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 1 & 3 & 1 \end{bmatrix} \}.$$

(b) Find a basis for the column space of A

SOLUTION: Since, in the reduced row echelon form, the pivots occur in columns 1, 3 and 4, we take columns 1, 3 and 4 of the original matrix A:

$$S = \left\{ \begin{bmatrix} 2 \\ -1 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ -6 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ -6 \\ 1 \end{bmatrix} \right\}.$$

(c) Find a basis for the nullspace of A

SOLUTION: First, let us write down all solutions to the homogeneous linear system $A\mathbf{x} = \mathbf{0}$. The free variables are $x_2 = r$, $x_5 = s$ and $x_6 = t$. The general form of a solution is then

$$\begin{bmatrix} 2r - s & r & -t & -3s - t & s & t \end{bmatrix}.$$

We obtain our three basis vectors by taking $r = 1, s = 0, t = 0$, then $r = 0, s = 1, t = 0$ and finally $r = 0, s = 0, t = 1$ to obtain:

$$S = \left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \\ -3 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -1 \\ -1 \\ 0 \\ 1 \end{bmatrix} \right\}.$$