Linear Algebra
C Term, Sections C01-C04
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Sample Solutions – Quiz 5

In each of the following cases, determine whether or not W is a subspace of the vector space P_3. Answer "YES" or "NO" and give sufficient justification for your answer.

(a)

$$W = \{p(t) : p(t) = a_3t^3 + a_2t^2 + a_1t + a_0 \text{ where } a_0a_1 = a_0a_2\}$$

SOLUTION: NO, this is **not** a subspace of P_3. For example, $p(t) = t^2$ lies in W since it has $a_0 = 0$ and $q(t) = t^2 + t + 1$ lies in W since it has $a_1 = a_2$, but $p(t) + q(t) = 2t^2 + t + 1$ has $a_0a_1 = 1$ and $a_0a_2 = 2$ so does not belong to W. This shows that W is not closed under addition.

(b)

$$W = \{p(t) : p(t) = a_3t^3 + a_2t^2 + a_1t + a_0 \text{ where } a_1 = a_2\}$$

SOLUTION: YES, this **is** a subspace of P_3. Let $p(t) = a_3t^3 + a_2t^2 + a_1t + a_0$ and $q(t) = b_3t^3 + b_2t^2 + b_1t + b_0$. Then

$$p(t) + q(t) = (a_3 + b_3)t^3 + (a_2 + b_2)t^2 + (a_1 + b_1)t + (a_0 + b_0)$$

and, given that $a_1 = a_2$ and $b_1 = b_2$, it follows that $a_1 + b_1 = a_2 + b_2$ showing that $p(t) + q(t)$ belongs to W. Next, let k be any scalar. Then

$$kp(t) = (ka_3)t^3 + (ka_2)t^2 + (ka_1)t + (ka_0)$$

and $a_1 = a_2$ implies $ka_1 = ka_2$ showing $kp(t)$ belongs to W. So W is closed under both addition and scalar multiplication.

(c)

$$W = \{p(t) : p(t) = a_3t^3 + a_2t^2 + a_1t + a_0 \text{ where } a_0 = 0\}$$

SOLUTION: YES, this **is** a subspace of P_3. Let $p(t) = a_3t^3 + a_2t^2 + a_1t$ and $q(t) = b_3t^3 + b_2t^2 + b_1t$. Then

$$p(t) + q(t) = (a_3 + b_3)t^3 + (a_2 + b_2)t^2 + (a_1 + b_1)t$$

also has zero constant term and is therefore in W. Similarly

$$kp(t) = (ka_3)t^3 + (ka_2)t^2 + (ka_1)t$$

has zero constant term whenever $p(t)$ does, so $kp(t)$ belongs to W.