Linear Algebra
C Term, Sections C01-C04
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**Sample Solutions – Quiz 1**

Consider the following system of linear equations:

$$\begin{array}{rrrcr} x_1 & +2x_2 & -2x_3 & = & 9 \\ 3x_1 & +6x_2 & +x_3 & = & 34 \\ -2x_1 & -4x_2 & +5x_3 & = & -17 \end{array}$$

**1.)** Write down the augmented matrix corresponding to this system.

*Solution:*

$$[A|b] = \left[\begin{array}{rrr|r} 1 & 2 & -2 & 9 \\ 3 & 6 & 1 & 34 \\ -2 & -4 & 5 & -17 \end{array}\right]$$

**2.)** Perform Gaussian elimination on this matrix to obtain a matrix in **reduced row echelon form**.

*Solution:*

$$\begin{array}{rcl} [A|b] & \sim & \left[\begin{array}{rrr|r} 1 & 2 & -2 & 9 \\ 0 & 0 & 7 & 7 \\ 0 & 0 & 1 & 1 \end{array}\right] \begin{array}{l} R1 \\ R2 - 3 \cdot R1 \\ R3 + 2 \cdot R1 \end{array} \\ & \sim & \left[\begin{array}{rrr|r} 1 & 2 & 0 & 11 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} R1 + 2R3 \\ R3 \\ R2 - 7 \cdot R3 \end{array} \end{array}$$

This last matrix is in r.r.e.f.

**3.)** Using the result of (2) above, find all solutions to the original linear system.

*Solution:* The solutions are

$$\begin{array}{rcl} x_1 & = & 11 - 2r, \\ x_2 & = & r, \\ x_3 & = & 1 \end{array}$$

where $r$ can be any number.