Linear Algebra
C Term, Sections C01-C04
W. J. Martin
February 14, 2003

Linear Algebra Assignment 6

DUE DATE: Friday, February 21, 4pm. Submit in class or to MA2071 mail slot in SH108.

N.B. No late assignments will be accepted for credit.

N.B. Keep in mind Professor Martin's rules for completing assignments (reproduced on the back of this sheet).

Please complete the following four problems:

1. Exercise #14 on page 273.

2. Exercise #T.10 on page 274.

3. Suppose
$$S = \{\mathbf{v}_1, \ldots, \mathbf{v}_k\} \qquad \text{and} \qquad T = \{\mathbf{w}_1, \ldots, \mathbf{w}_\ell\}$$
are linearly independent subsets of the vector space $\mathbb{R}^n$. Devise an algorithm, based on Gauss-Jordan reduction, which finds a basis for
$$W = \mathsf{span}(S) \cap \mathsf{span}(T).$$
(Here, $\cap$ denotes the intersection of two sets, the set of all vectors that belong to both.)
HINT: The vectors in W are determined by the solutions to the equation
$$c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \cdots + c_k\mathbf{v}_k = d_1\mathbf{w}_1 + d_2\mathbf{w}_2 + \cdots + d_\ell\mathbf{w}_\ell$$

4. Consider the matrix
$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -34 & -9 & 8 & 12 \\ -18 & -6 & 7 & 6 \\ -22 & -6 & 4 & 9 \end{bmatrix}.$$
We would like to find all vectors $\mathbf{x}$ which satisfy $A\mathbf{x} = \theta\mathbf{x}$ for various real numbers θ. To find all such $\mathbf{x}$, it is enough to find a basis for the nullspace of the matrix $A - \theta I$.

(a) (the case $\theta = 1$): Find a basis for $\mathsf{nullspace}(A - I)$.

(b) (the case $\theta = 2$): Find a basis for $\mathsf{nullspace}(A - 2I)$. *[NOTE: the answer is a bit strange.]*

(c) (the case $\theta = 3$): Find a basis for $\mathsf{nullspace}(A - 3I)$.

Professor Martin's Rules for Linear Algebra Assignments:

- Write neatly, using correct English.
- Use **only one side** of each sheet of paper. Ink on the back of the page deteriorates the readability of what is on the front.
- Explain your steps. A correct answer with no explanation will earn a grade of zero.
- Use a staple when you submit more than one sheet and want them all back. There is a stapler for public use in the Mathematical Sciences Department Office (SH108).