Linear Algebra
C Term, Sections C01-C04
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Linear Algebra Assignment 3

DUE DATE: Friday, January 31, 4pm. Submit in class or to MA2071 mail slot in SH108.

N.B. No late assignments will be accepted for credit.

N.B. Keep in mind Professor Martin's rules for completing assignments (reproduced on the back of this sheet).

Please complete the following four problems:

1. For each of the following functions, decide whether or not it is a linear transformation. If so, show that the defining properties hold for all inputs. If it is **not** a linear transformation, demonstrate (*with specific examples*) a property which fails.

   (a) $L : \mathbb{R}^2 \to \mathbb{R}^3$ via
   $$L\left(\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}\right) = \begin{bmatrix} u_1 \\ u_2 \\ u_1 u_2 \end{bmatrix}$$

   (b) $L : \mathbb{R}^2 \to \mathbb{R}^2$ via
   $$L\left(\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}\right) = \begin{bmatrix} u_1 + u_2 \\ u_1 - u_2 \end{bmatrix}$$

   (c) $L : \mathbb{R}^4 \to \mathbb{R}^2$ via
   $$L\left(\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}\right) = \begin{bmatrix} 2u_4 \\ 0 \end{bmatrix}$$

   (d) $L : \mathbb{R}^2 \to \mathbb{R}^2$ via
   $$L\left(\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}\right) = \begin{bmatrix} 2u_1 + 5 \\ u_1 + u_2 \end{bmatrix}$$

2. For each of the following linear transformations, write down the standard matrix representing L.

   (a) $L : \mathbb{R}^2 \to \mathbb{R}^3$ via
   $$L\left(\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}\right) = \begin{bmatrix} u_1 - u_2 \\ 2u_2 \\ 5u_1 - u_2 \end{bmatrix}$$

(b) $L : \mathbb{R}^2 \to \mathbb{R}^2$ via

$$L\left(\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}\right) = \begin{bmatrix} 2u_2 \\ 2u_1 \end{bmatrix}$$

(c) $L : \mathbb{R}^4 \to \mathbb{R}^2$ via

$$L\left(\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}\right) = \begin{bmatrix} u_1 + 7u_3 - 2u_4 \\ 0 \end{bmatrix}$$

3. Exercise #T.5 on pages 213.

4. Exercise #T.8 on page 214.

PROFESSOR MARTIN'S RULES FOR LINEAR ALGEBRA ASSIGNMENTS:

- Write neatly, using correct English.
- Use **only one side** of each sheet of paper. Ink on the back of the page deteriorates the readability of what is on the front.
- Explain your steps. A correct answer with no explanation will earn a grade of zero.
- Use a staple when you submit more than one sheet and want them all back. There is a stapler for public use in the Mathematical Sciences Department Office (SH108).