Linear Algebra
C Term, Sections C01-C04
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Linear Algebra Assignment 1

Due Date: Friday, January 17, 4pm. Place in mail slot labelled "MA2071 Assignments" in Room SH108. Alternatively, assignments may be given to Dr. Lui in class. **Do Not** place assignments in Dr. Martin's mail slot under any circumstances.

N.B. No late assignments will be accepted for credit. Sample solutions will typically be available as soon as the deadline passes. After this, any work not yet turned in will receive a grade of zero.

Professor Martin's Rules for Linear Algebra Assignments:

- Write neatly, using correct English.
- Use **only one side** of each sheet of paper. Ink on the back of the page deteriorates the readability of what is on the front.
- Explain your steps. A correct answer with no explanation will earn a grade of zero.
- Use a staple when you submit more than one sheet and want them all back. There is a stapler for public use in the Mathematical Sciences Department Office (SH108).

Please complete the following four problems:

1. Not all algebraic systems are as intuitive as the real numbers. For example, if $A = \begin{bmatrix} 6 & 9 \\ -4 & -6 \end{bmatrix}$, then $A^2 := AA = 0$. Systematically determine all 2×2 matrices $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ which satisfy $A^2 = 0$.

2. Do problem # 26 on page 9. Explain your solution in complete sentences.

3. Two $n \times n$ matrices A and B are said to **commute** if $AB = BA$. This seldom happens at random but often plays an important role in problems in linear algebra. By solving a linear system, find all values a, b, c, d, e, f for which the 3×3 matrix

$$A = \begin{bmatrix} a & 3 & b \\ 3 & c & d \\ e & 3 & f \end{bmatrix} \qquad \text{commutes with the matrix} \qquad B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 0 & 0 \\ 3 & 0 & 9 \end{bmatrix}.$$

Note that you may use shortcuts (such as eliminating a variable early based on obvious restrictions), but you are expected to explain your solution in English sentences.

4. *In this course, we will discuss vector spaces other than the obvious one consisting of all* $n \times 1$ *matrices. For example, the set of all continuous functions defined on a given real interval is a vector space.*

In this problem. we will analyze the following "dot product" on the space of all continuous functions on $[-\pi, \pi]$:

$$\langle f, g \rangle := \int_{-\pi}^{\pi} f(x)g(x)\,dx.$$

Two vectors in a vector space are **orthogonal** if their dot product is zero. By computing the above dot product for each of the six possible pairs, determine which pairs among the following functions are orthogonal pairs:

(a) $a(x) = \sin x$ for $-\pi \le x \le \pi$

(b) $b(x) = \cos x$ for $-\pi \le x \le \pi$

(c) $c(x) = x$ for $-\pi \le x \le \pi$

(d) $d(x) = 1$ for $-\pi \le x \le \pi$

NOTE: In signal processing, signals represented by orthogonal functions can be easily separated by a filter. This is how a cell phone network can allow multiple users on the same frequency.