Linear Algebra
C Term, Sections C01-C04
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February 18, 2002

Solutions — Linear Algebra Quiz 6

Find a basis for the null space of the following matrix. Show your work!

$$A = \begin{bmatrix} 2 & -4 & -1 & -1 \\ 3 & -6 & -1 & 0 \\ -2 & 4 & 0 & -2 \\ 1 & -2 & 4 & 13 \end{bmatrix}$$

Solution: First, we row reduce the augmented matrix

$$[A|\mathbf{0}] = \left[\begin{array}{rrrr|r} 2 & -4 & -1 & -1 & 0 \\ 3 & -6 & -1 & 0 & 0 \\ -2 & 4 & 0 & -2 & 0 \\ 1 & -2 & 4 & 13 & 0 \end{array}\right] \sim \left[\begin{array}{rrrr|r} 1 & -2 & 4 & 13 & 0 \\ 0 & 0 & -13 & -39 & 0 \\ 0 & 0 & 8 & 24 & 0 \\ 0 & 0 & -9 & -27 & 0 \end{array}\right] \begin{array}{r} (R4) \\ (R2)-3(R4) \\ (R3)+2(R4) \\ (R1)-2(R4) \end{array}$$

$$\sim \left[\begin{array}{rrrr|r} 1 & -2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} (R1)+\frac{4}{13}(R2) \\ \frac{-1}{13}(R2) \\ (R3)+\frac{8}{13}(R2) \\ (R4) - \frac{9}{13}(R2) \end{array}$$

The solutions to this homogeneous linear system may therefore be expressed in terms of two free parameters:

$$x_1 = 2r - s, \qquad x_2 = r, \qquad x_3 = -3s, \qquad x_4 = s$$

Setting $r = 1$ and $s = 0$, we obtain one solution vector

$$\mathbf{v}_1 = [2 \;\; 1 \;\; 0 \;\; 0]^T$$

and setting $s = 1$, $r = 0$, we obtain another:

$$\mathbf{v}_2 = [-1 \;\; 0 \;\; -3 \;\; 1]^T$$

These are clearly linearly independent and so we have a basis

$$S = \{\mathbf{v}_1, \mathbf{v}_2\}$$

for the null space of A.