Linear Algebra
C Term, Sections C01-C04
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Solutions — Linear Algebra Quiz 4

Consider the following structure. We have a set V of vectors described by

$$V = \left\{ \begin{pmatrix} a \\ b \end{pmatrix} : \; a > 0, \; b > 0 \right\}$$

with the following operations

$$\begin{pmatrix} a \\ b \end{pmatrix} \oplus \begin{pmatrix} c \\ d \end{pmatrix} = \begin{pmatrix} ac \\ bd \end{pmatrix}$$

for $\begin{pmatrix} a \\ b \end{pmatrix}$ and $\begin{pmatrix} c \\ d \end{pmatrix}$ in V, and

$$r \odot \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} ra \\ rb \end{pmatrix}$$

for any real number r and any $\begin{pmatrix} a \\ b \end{pmatrix}$ in V.

1.) [2 points] Does the set V form a vector space under these operations?
(/ **NO**) (circle one)

2.) [3 points] If you answered "YES" to Question 1, then write down the zero vector for this vector space with a brief explanation.

If you answered "NO" to Question 1, demonstrate some property of vector spaces which this structure fails to satisfy. Be specific: use actual numbers for your sample vectors.

Solution: **NO**, this is not a vector space. For example, the set V fails to be closed under scalar multiplication. Here is a specific example. Let $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Then clearly $\mathbf{v} \in V$. But $(-1) \odot \mathbf{v} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$ does **not** belong to V. So property (β) fails.

SOME NOTES: The set V does satisfy properties (α) and (a)–(d). (This is interesting!) But, even if (β) were satisfied, properties (e) and (f) in the definition of a vector space. (Can you find some specific values to show this?)