Linear Algebra
C Term, Sections C01-C04
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Solutions – Linear Algebra Quiz 3

For each of the following functions T, determine whether or not T is a linear transformation. In addition to an asnwer of "YES" or "NO", give sound reasons for your answer.

(a) $T : R^3 \to R^2$ via

$$T\left(\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}\right) = \begin{bmatrix} u_1 - 2u_2 \\ u_1 + u_3 - 4 \end{bmatrix}$$

Solution: **NO**, this is not a linear transformation. For example,

$$T(\mathbf{0}) = \begin{bmatrix} 0 \\ -4 \end{bmatrix} \neq \mathbf{0}.$$

(b) $T : R^2 \to R^2$ via

$$T\left(\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}\right) = \begin{bmatrix} u_1 - 2u_2 \\ u_1 + 2u_2 \end{bmatrix}$$

Solution: **YES**, this is a linear transformation. For if $\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$, then

$$T(\mathbf{u}+\mathbf{v}) = T\left(\begin{bmatrix} u_1 + v_1 \\ u_2 + v_2 \end{bmatrix}\right) = \begin{bmatrix} (u_1+v_1) - 2(u_2+v_2) \\ (u_1+v_1) + 2(u_2+v_2) \end{bmatrix} =$$

$$\begin{bmatrix} u_1 - 2u_2 \\ u_1 + 2u_2 \end{bmatrix} + \begin{bmatrix} v_1 - 2v_2 \\ v_1 + 2v_2 \end{bmatrix} = T(\mathbf{u}) + T(\mathbf{v})$$

and, if r is any real number,

$$T(r\mathbf{u}) = T\left(\begin{bmatrix} ru_1 \\ ru_2 \end{bmatrix}\right) = \begin{bmatrix} ru_1 - 2ru_2 \\ ru_1 + 2ru_2 \end{bmatrix} = r \begin{bmatrix} u_1 - 2u_2 \\ u_1 + 2u_2 \end{bmatrix} = r\, T(\mathbf{u})$$

(c) (Typographical error corrected) $T : R^2 \to R^3$ via

$$T\left(\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}\right) = \begin{bmatrix} u_1 - 2u_2 \\ u_2^2 \\ u_1 + 4u_2 + u_2 \end{bmatrix}$$

Solution: **NO**, this is not a linear transformation. For example, let

$$\mathbf{u} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Then

$$T(\mathbf{u}) = \begin{bmatrix} -2 \\ 1 \\ 5 \end{bmatrix}$$

but

$$T(\mathbf{u} + \mathbf{u}) = T\left(\begin{bmatrix} 0 \\ 2 \end{bmatrix}\right) = \begin{bmatrix} -4 \\ 4 \\ 10 \end{bmatrix} \neq T(\mathbf{u}) + T(\mathbf{u})$$