Linear Algebra   **PRACTICE ONLY – DO NOT HAND IN**
C Term, Sections C01-C04
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Linear Algebra Quiz 2

Find the inverse of the following matrix, if it exists:

$$A = \begin{pmatrix} -\frac{1}{6} & \frac{1}{3} & -\frac{1}{6} \\ -\frac{5}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{3}{2} & -\frac{1}{3} & -\frac{1}{6} \end{pmatrix}$$

SOLUTION:

We row reduce the block matrix $[A|I]$:

$$[A|I] \sim \begin{pmatrix} -1 & 2 & -1 & 6 & 0 & 0 \\ -5 & 1 & 1 & 0 & 3 & 0 \\ 9 & -2 & -1 & 0 & 0 & 6 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & -2 & 1 & -6 & 0 & 0 \\ 0 & -9 & 6 & -30 & 3 & 0 \\ 0 & 16 & -10 & 54 & 0 & 6 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & -2 & 1 & -6 & 0 & 0 \\ 0 & 1 & -2/3 & 10/3 & -1/3 & 0 \\ 0 & 0 & 2/3 & 2/3 & 16/3 & 6 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 0 & 0 & 1 & 2 & 3 \\ 0 & 1 & 0 & 4 & 5 & 6 \\ 0 & 0 & 1 & 1 & 8 & 9 \end{pmatrix}$$

(Note that some steps were combined.) We arrive at the conclusion

$$A^{-1} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 1 & 8 & 9 \end{pmatrix}$$