Linear Algebra
C Term, Sections C01-C04
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Linear Algebra Assignment 7

OPTIONAL (see note)

DUE DATE: Wednesday, February 27, noon. Deliver to your conference PLA. No late assignments will be accepted for credit.

N.B. Keep in mind Professor Martin's rules for completing assignments.

Please complete the following four problems:

1. Complete Exercise # 14 on p304.

2. Suppose A is a 4×4 matrix with eigenvalues $4, 3, 2, 1$.

   (a) What are the eigenvalues of $5A$? Explain.

   (b) What are the eigenvalues of A^2? Explain.

   (c) What are the eigenvalues of A^T? Explain.

3. Let $A = \begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}$.

   (a) Find all eigenvalues of A and an associated eigenvector for each.

   (b) Find a diagonal matrix D which is similar to A.

   (c) Using part (b), compute A^5. Explain.

4. Let $\mathbf{v}_1 = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

   (a) Find all 3×3 symmetric matrices A for which $\mathbf{v}_1$ is an eigenvector associated to eigenvalue $\lambda_1 = -1$ and $\mathbf{v}_2$ is an eigenvector associated to eigenvalue $\lambda_2 = 1$. What is the remaining eigenvalue of A?

   (b) Explain why there is no such matrix if we replace $\mathbf{v}_2$ by $\mathbf{w}_2 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$.

NOTE: In the syllabus, we have agreed to take the best five assignment grades to obtain a 25% portion of the final grade in the course. At the request of several students, I am offering this seventh assignment to anyone who wishes to improve their homework average. So I will now take the best 5 grades out of 7.