Matrices & Linear Algebra I
C Term, Sections C01-C04
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Week 3 – Conference Sections

Progress

Since last Wednesday, we have covered Sections 1.6, 3.1–3.3 and 4.1 in class. We learned a bit of the theory of inverses, we learned how to compute an inverse, we went briskly through the theory of determinants and now we are looking at vectors in 2 and 3 dimensions.

I have not yet made it clear exactly what the students will be expected to know regarding determinants. My primary interest in determinants is in obtaining eigenvalues.

Assignment 2

Please collect Assignment 2 at the beginning of class. Hand out the sample solutions. No late assignments are to be accepted for credit.

After conference has ended, place the assignments in your large envelope. Place the envelope in the TA's mailbox (Hantao Mai, SH108).

What to Demonstrate

1. Show them how to invert a matrix using Gauss-Jordan reduction. Be sure to document all the steps.

   Here are two nice examples:

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}.$$

   We form the augmented matrix $[A|I]$ and row reduce. On the left-hand side, we have

$$A \sim \begin{pmatrix} 1 & 2 & 3 \\ 0 & -3 & -6 \\ 0 & -6 & -12 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{pmatrix}$$

So we can stop here and say that A is singular.

In the second example, let's see the determinant show up in the inverse. Let

$$A = \begin{pmatrix} 0 & 1 & -2 \\ 1 & 2 & 4 \\ 2 & 4 & 3 \end{pmatrix}.$$

Then you tack on the identity and row reduce to obtain

$$A^{-1} = \frac{1}{5}\begin{pmatrix} -10 & -11 & 8 \\ 5 & 4 & -2 \\ 0 & 2 & -1 \end{pmatrix}.$$

2. Now show how to compute the determinant as a by-product of Gauss-Jordan reduction. Recall the rules

   - If B is obtained from A by swapping two rows, then $\det(B) = -\det(A)$
   - If B is obtained from A by multiplying a row by a non-zero constant c, then $\det(B) = c \cdot \det(A)$
   - If B is obtained from A by adding a multiple of one row to another, then $\det(B) = \det(A)$

   So go back to the above examples and compute determinants. Let $\det(A) = z$ (unknown). Write "z" next to the original matrix. Now apply the above rules to compute the determinant of each matrix in the sequence as a function of z. E.g., in the first example, our first step is to subtract multiples of the first row from the second and third. So the second matrix I gave above will still have z as its determinant. Keep going. At the end you either end up with a row of zeros (in which case the determinant is zero) or you arrive at the identity matrix (which has determinant one). So at this point you can solve for z.

   This method is **much faster** than the direct computation of the determinant if the matrix is 6×6 or larger.

3. Entertain any questions from the Reading and Exercises (available on the web).

4. If there is any time left, explain one or two proofs from Section 1.6.