Bridge to Higher Math
D Term 2011
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MA197X Problem Set 4

Instructions: Please review the rules on the presentation of assignments in the course. Then complete the following ten problems and submit the solutions, inside your portfolio folder, by Wednesday, April 13th.

For each of the following problems, first state the problem precisely in English and then give a proper proof of the statement using English sentences. Be sure to include the correct problem numbers for recording purposes.

27. For all sets A, B, C and D, and any functions f, g and h, if $f : A \to B$, $g : B \to C$ and $h : C \to D$, then
$$h \circ (g \circ f) = (h \circ g) \circ f.$$

28. For all sets A and B and for any function $f : A \to B$,
$$I_B \circ f = f \qquad \text{and} \qquad f \circ I_A = f.$$

29. For all sets A, B and C and any functions $f : A \to B$ and $g : B \to C$
(a) if f and g are both injective, then $g \circ f$ is also injective.
(b) if $g \circ f$ is an injection, then f is also an injection.
(c) The following is FALSE: For all sets A, B and C and any functions $f : A \to B$, if $g \circ f$ is an injection, then g is also an injection.

30. For all sets A, B and C and any functions $f : A \to B$ and $g : B \to C$
(a) if f and g are both surjective, then $g \circ f$ is also surjective.
(b) if $g \circ f$ is an surjection, then g is also a surjection.
(c) The following is FALSE: For all sets A, B and C and any functions $f : A \to B$, if $g \circ f$ is an surjection, then f is also a surjection.

31. The following proposition is FALSE: *For all sets A and B and all functions $f : A \to B$ and $g : B \to A$, if $g \circ f = I_A$, then $f \circ g = I_B$.*

32. For all sets A and B and all $f : A \to B$ if f is both one-to-one and onto, then $(f^{-1})^{-1} = f$.

33. For all sets A and B and all $f : A \to B$ and $g : B \to A$, if f and g are both bijections and $g \circ f = I_A$ then $g = f^{-1}$.

34. Consider $A = \{1,2,3\}$ and $B = \{1,2,3,4\}$ and $f, g, h \subseteq A \times B$ defined by

$$f = \{(1,3),(2,4),(2,2),(3,3)\}, g = \{(1,4),(2,1),(3,2),(2,1)\}, h = \{(2,1),(3,1),(2,1),(3,1)\}.$$

(a) Decide which of these are functions from A to B. Concisely explain. **(b)** Do the same for all nine possible compositions of these relations $f \circ f$, $f \circ g$, $f \circ h$, $g \circ f$, $g \circ g$, $g \circ h$, $h \circ f$, $h \circ g$, $h \circ h$ (where the definition of $\circ$ is given in Problem Set 3). Concisely explain.

35. Let A, B, C and D be sets and let $f : A \to C$ and $g : B \to D$. Define a function $H : A \times B \to C \times D$ by

$$H(a,b) = (f(a), g(b))$$

for all $(a,b) \in A \times B$.
(a) Prove: if f and g are both one-to-one, then H is also one-to-one.
(b) Disprove: if f and g are both onto, then H is also onto.

36. Let n be a positive integer and consider $f : \mathbb{R}^n \to \mathbb{R}$ given by

$$f(x_1, x_2, \ldots, x_n) = x_1 + x_2 + \cdots + x_n.$$

Prove that f is onto, but that f is one-to-one if and only if $n = 1$.