Bridge to Higher Math
D Term 2011
W. J. Martin
March 16, 2011

MA197X Problem Set 1

Instructions: Please first read the rules on the presentation of assignments in the course. Then complete the following six problems and submit the solutions, inside your portfolio folder, by Tuesday, March 22nd.

For each of the following problems, first state the problem precisely in English and then give a proper proof of the statement using English sentences.

1. The following hold for any integers:
   (a) If $a|b$ and $b|a$, then $b = \pm a$;
   (b) If a and b are positive and $a|b$, then $a \leq b$;
   (c) $a|a$;
   (d) If a and b are positive and $a|b$ and $b|a$, then $b = a$;
   (e) If $a|b$ and $b|c$, then $a|c$.
2. The following hold for any integers:
   (a) If $a|b$ then $a|bx$ for any integer x;
   (b) If $a|b$ and $a|c$, then $a|(bx + cy)$ for any integers x and y;
   (c) If $a|b$ and $c|d$, then $ac|bd$.
3. The following are all false:
   (a) For all integers a, b, c, if $a|bc$ then either $a|b$ or $a|c$;
   (b) For all integers a, b, c, d, if $a|b$ and $c|d$ then $(a + c)|(b + d)$;
   (c) For all integers a, b, c, if $a \nmid b$ and $a \nmid c$ then $a \nmid bc$;
   (d) For all integers a, b, c, if $a \nmid b$ and $b \nmid c$ then $a \nmid c$.
4. If n is an odd integer, then $8|(n^2 - 1)$.
5. If p is prime and $p|ab$, then $p|a$ or $p|b$.
6. Consider the following two conjectures:
   Conjecture A: For every integer $n > 1$, there exists a prime number between n and n^2.
   Conjecture B: For every integer $n > 1$, there exists a prime number between n and $2n$.
   While we may not currently know if either of these is true, one implies the other. Figure out which implies which and prove this implication.