MA196x Syllabus

Knowing with Certainty: Proofs in Contemporary Mathematics

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or by appointment
. . . or, just drop in

MA 196x – D01, D Term 2009
Knowing with Certainty
Text: "Write Your Own Proofs: In Set Theory and Discrete Mathematics by A. Babich and L. Person
Meetings: 1:00–1:50 MTRF

Course web page: http://users.wpi.edu/∼martin/TEACHING/1960/

The goal of this course is to introduce the student to mathematical thinking. The defining characteristic of mathematics as a science is its reliance upon formal logical proof. Mathematics is not an experimental science where "laws" are the result of repeated observations. Every law in mathematics is either an axiom (a basic assumption) or something that has been rigorously derived. We will explore this concept of proof on two levels, at the same time.

Our primary goal is to develop in each student a familiarity with the mechanics of basic mathematical proofs. Advanced proofs are characterized by their creativity, by the connections they draw between seemingly unrelated facts and concepts, and by their novel application of known theory. But before the student can ascend to these intriguing heights, s/he must first know what makes a correct proof and how to distinguish proper logical arguments from fallacious ones (or from just plain bullshit). The student must first learn the difference between a hypothesis and a conclusion, between a universal and an existential quantifier, between direct and indirect proofs, and so on. These mechanics of proof form the core of the first part of the course.

A course of this sort, however important and however fundamental to the student's success in higher mathematics, can be very dry at times. Students are full of imagination and are eager to see the bigger picture and to apply the knowledge they've obtained. The second component of this course is to consider the changing nature of proof in modern mathematics. Some who know little of the subject assume that mathematics is "old science", that all of the theorems needed have been proven for more than a century and that there is nothing left to discover. This is quite far from the truth. Mathematics is alive! It is thriving, evolving, developing year by year, with new problems solved, new connections discovered, and new questions emerging, defying the mathematical community to answer them. When mathematicians answer these questions, the answers are not just good for a few years, or decades. These are proofs and, when correct, the answers they give are true *forever*! In this course, students will engage in activities and take on projects which allow them to explore this living, breathing thing called mathematics.

A TYPICAL MEETING

We have 28 meetings together and I do not want to constrain us by prescribing where we will be in the main text on any given day or exactly how the class will be conducted.

All students are expected to arrive at class **fully prepared** to participate; any student can be called upon at any time to lead the class.

While I expect we will have a few lecture periods, my intention is that this will be the exception rather than the rule. In a given class, we may do any of the following:

- discuss the text;
- have one or more of us present material from the text;
- solve problems in groups;
- discuss contemporary issues in mathematics.

GRADING SCHEME

Classroom Participation:	20 %
Homework (best 40 problems):	50 %
Mathematics Activities:	10 %
Team Project:	20 %

There will be no tests.

GRADES

Grades will be assigned on a curve according to Dr. Martin's overall grade to the class as a group. These grades will satisfy the following conditions:

C Any student with at least a 70% overall average will receive a C or higher.

B Any student with at least a 80% overall average will receive a B or higher.

A Any student with at least a 90% overall average will receive an A.

CLASSROOM PARTICIPATION

Mathematics is not a spectator sport. A student must be actively involved in the learning of mathematics and in the development of the course to truly benefit from it. Therefore all students are expected to participate.

In every class, all students will be graded on their participation in that class. This can take a number of forms, such as presenting at the blackboard, asking questions, answering questions, helping a classmate, participating in discussions, providing written solutions to problems. (Absences from particular class meetings – when properly justified – will be handled on an individual basis.)

HOMEWORK

Each student should maintain a portfolio of graded homework problems. Each problem will be graded on a scale of one to five, reflecting both correctness and clarity of the solution and difficulty of the problem. At the end of the course, the student shall submit his or her portfolio with the forty problems that the student wants included in his or her final grade. Each student must have completed at least 30 problems by April 17 and no problems will be accepted for credit after May 1.

ACTIVITIES

As we all know, mathematics is alive, even today. Even on the WPI campus, there are exciting mathematical activities open to the interested student, in any given academic year. For example, Math Hour is a weekly event where students interested in mathematics gather to discuss problems, ideas, puzzles and games.

Each student in the course is required to attend

- The **Levi Conant Lecture:** Dr. Brian Conrey, Executive Director of the American Institute of Mathematics, will speak on the Riemann Hypothesis, Monday, March 30, 2009 at 4:00pm in HL116. Every student is expected to attend.
- The **Spring 2009 Meeting** of the American Mathematical Society will be held on the WPI campus Saturday-Sunday April 25-26, 2009. Every student is expected to attend at least one of the 100+ talks to be given at the conference and to write a short report on the talk, interviewing the speaker, if necessary. Details to follow.
- Weekly research colloquia in the Department of Mathematical Sciences — any student who misses one of the above two assigned events may, with the instructor's permission, write a report on a selected colloquium talk. Colloquia are typically held at 11am on Fridays in SH203.

- Mathematics Awareness Month activities. All students are expected to take part in appropriate advertised activities. (I will give you more details later in the term.)

PROJECT

The capstone experience for the course is a team project worth 20% of your grade. The goal of the project is to examine, at a level appropriate to your mathematical training, an issue or problem at the forefront of mathematics today. The project will culminate in a 30-page typeset report together with an oral team presentation in class at the end of D Term.

Here are some possible topics for the project:

- The proof of Fermat's Last Theorem
- The Clay Millenium Prizes
- The Classification of Finite Simple Groups
- Computer-Generated Proofs and Conjectures
- Logic Programming
- Proof versus experimentation in mathematical physics
- The Kepler Conjecture
- The Four-Color Theorem

IMPORTANT NOTES

- Collaboration with classmates is *usually* strongly encouraged. But in the case of homework problems to be submitted for homework credit, consultation with classmates should be limited to the solicitation of critiques only, and only after a reasonable first draft of your solution has been completed.

STUDENTS WITH DISABILITIES

If you need course adaptations or accommodations because of a disability, or if you have medical information to share with me, please make an appointment with me as soon as possible. My office location and hours are listed at the top of this syllabus. If you have not already done so, students with disabilities, who believe that they may need accommodations in this class, are encouraged to contact the Disability Services Office (DSO), as soon as possible to ensure that such accommodations are implemented in a timely fashion. The DSO is located in Daniels Hall, (508) 831-5235.

ACADEMIC INTEGRITY

Please read the Student Guide to Academic Integrity at WPI and all its pages. For example, the page What Constitutes Academic Dishonesty? – found here:
`http://www.wpi.edu/Pubs/Policies/Honesty/Students/constitutes.html`
– gives some examples of academic dishonesty; i.e. acts that interfere with the process of evaluation by misrepresenting the relation between the work being evaluated (or the resulting evaluation) and the student's actual state of knowledge.

Each student is responsible for familiarizing him- or herself with academic integrity issues and policies at WPI. All suspected cases of dishonesty will be fully investigated.