Calculus II
D Term
W. J. Martin
March 20, 2001

SOME PROPERTIES OF THE DEFINITE INTEGRAL

This handout lists the most important properties of the definite integral. Proofs are omitted.

Endpoints

1. $\int_a^a f(x)\,dx = 0$ for any function f and any real number a such that $f(a)$ is defined.

2. ("IAP") If f is integrable on $[a,b]$, $[a,c]$ and $[c,b]$, then (regardless of the order of the three numbers) we have

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx \; + \; \int_c^b f(x)\,dx$$

3. If $a \leq b$ and f is integrable on $[a,b]$, then

$$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$$

Linearity

4. If c is a constant and f is integrable on $[a,b]$, then

$$\int_a^b cf(x)\,dx = c\int_a^b f(x)\,dx$$

5. If both f and g are integrable on $[a,b]$, then

$$\int_a^b c\left[f(x)+g(x)\right]\,dx = \int_a^b f(x)\,dx + \int_a^b g(x)\,dx$$

6. If both f and g are integrable on $[a, b]$, then

$$\int_a^b c\,[f(x) - g(x)]\,dx = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$$

Comparison (Here, assume $a < b$ and that all integrals exist)

7. If $f(x) \geq 0$ for all x in the interval $[a, b]$, then

$$\int_a^b f(x)\,dx \geq 0$$

8. If $f(x) \geq g(x)$ for all x in the interval $[a, b]$, then

$$\int_a^b f(x)\,dx \geq \int_a^b g(x)\,dx$$

9. If $m \leq f(x) \leq M$ for all x in the interval $[a, b]$, then

$$m(b - a) \leq \int_a^b f(x)\,dx \leq M(b - a)$$

Symmetry

10. If f is an even function (i.e., $f(-x) = f(x)$ for all x) and both integrals exist, then

$$\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx$$

11. If f is an odd function (i.e., $f(-x) = -f(x)$ for all x) and the integral exists, then

$$\int_{-a}^{a} f(x)\,dx = 0$$