CIMPA Philippines
Semidefinite Prog. in Alg. Combin.
July 30, 2009

Student Problem Set 6

Instructions: Please solve these problems in today's problem session at 4:30 (and continue later, as needed) in groups and present the solutions on the black/whiteboards in the front of the room. You are encouraged to ask questions and to ask for help.

1. Prove that the irreducible representations of a finite abelian group have dimension 1.
2. Use the previous exercise and the magic formula $|G| = \sum d_R^2$ to prove that an abelian group G has $|G|$ irreducible representations.
3. We want to compute the irreducible representations of A_4.
   (a) Find a normal subgroup H of A_4 such that the quotient A_4/H is cyclic of order 3.
   (b) Construct 3 representations of degree 1 of A_4 from the ones of A_4/H.
   (c) Use the magic formula to have a guess for the degree of a missing irreducible representation of A_4.
   (d) Construct an irreducible representation of degree 3 of A_4 from the isometries of a regular polytope.
   (e) Conclude
4. Prove that $\langle \chi_V, \chi_V \rangle = 1$ if and only if V is irreducible.
5. For $V = P_k$, which denotes the subspace of $C(H_n)$ generated by the χ_y with $|y| = k$ compute $\langle \chi_V, \chi_V \rangle$ and use previous exercise to derive that P_k is irreducible. Here we are considering the action of $T \rtimes S_n$. Is P_k irreducible for the action of S_n ?
6. Prove Parseval formula in Frank talk:
$$\langle f, g \rangle = \sum_{u \in \{0,1\}^n} \hat{f}(u)\hat{g}(u)$$
7. Prove the formula in Frank talk:
$$\widehat{1_{A-y}}(u) = \widehat{1_A}(u)\chi_u(y)$$
8. Prove that, for the Krawtchouk polynomials K_k associated to the Hamming cube $\{0,1\}^n$,
$$\binom{n}{u} K_k(u) = \binom{n}{k} K_u(k)$$