CIMPA Philippines
Semidefinite Prog. in Alg. Combin.
July 29, 2009

Student Problem Set 5

Instructions: Please solve these problems in today's problem session at 4:30 (and continue later, as needed) in groups and present the solutions on the black/whiteboards in the front of the room. You are encouraged to ask questions and to ask for help.

1. Let E be the Euclidean space of real symmetric $n \times n$ matrices and let $\mathcal{K}$ be the intersection of the following two cones: the cone of positive semidefinite matrices and the cone of nonnegative matrices (i.e., those matrices in which each entry is at least zero). Determine the dual cone $\mathcal{K}^*$ for this cone and thereby verify Christine's formulation for the dual SDP for Lovász's ϑ' function of a graph.
2. For Γ the four-cycle, prove that $\alpha(\Gamma^n) \geq \alpha(\Gamma)^n = 2^n$ for every $n \geq 1$.
3. For Γ the four-cycle, verify that $\limsup_{n\to\infty} \alpha\left(\Gamma^n\right)^{1/n} = 2$.
4. For Γ the five-cycle (pentagon), show that $\alpha\left(\Gamma^2\right) = 5$.
5. Prove the second inequality in the Sandwich Theorem.
6. Use the Sandwich Theorem and an easy-to-obtain coloring to determine the chromatic number of the (complement of the) pentagon.
7. Determine the chromatic number of the Johnson graph $J(5, 2)$ (i.e., the complement of the Petersen graph). What does the Sandwich Theorem say in this case?
8. Prove that, for any positive number d, any subset of $\mathbb{R}^2$ that avoids the distance d must have density at most 0.268412.