Hey! You Can't Do That With My Code!

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and
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CIMPA-UNESCO-PHILIPPINES Research Summer School
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Outline

(T, M, S)-Nets

Resilient Functions

Fuzzy Extractors

First: The Omissions

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- the theory of **quantum error-correcting codes**
- The **PCP Theorem** in computational complexity theory: e.g. $NP = PCP_{1-\epsilon,\frac{1}{2}}[O(\log n), 3]$ (Håstad, 2001)

Part I: (T, M, S)-Nets

Using Codes to Estimate Integrals

If orthogonal arrays can be used to approximate Hamming space, can they also be used to approximate other spaces?

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- **2004+:** Improved bounds (Schmid/Schürer/Bierbrauer/Barg/Purkayastha/Trinker/Visentin)

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of volume q^{T-M} (i.e., with $d_1 + d_2 + \cdots + d_S = M - T$) contains exactly q^T points from $\mathcal{N}$.

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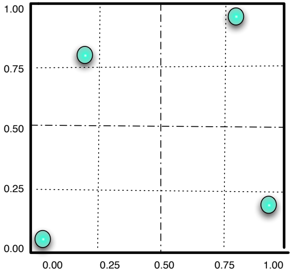
- 

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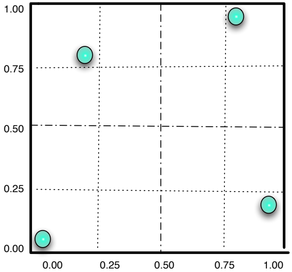

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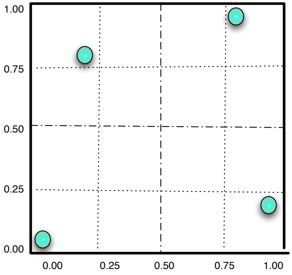
- 

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- **Lawrence/Mullen/Schmid:** $\exists(T, M, S)$-net in base $q \Leftrightarrow \exists OOA$ over $\mathbb{F}_q$ with q^m rows, $s = S$, $\ell = t = M - T$.

The Theorem of Mullen & Schmid and (indep.) Lawrence



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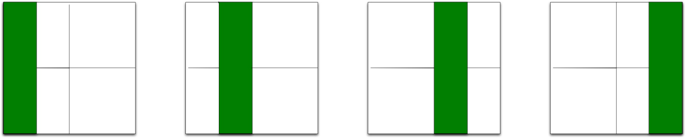
Idea of Proof

$$\mathcal{N} = \left\{ \left(\frac{0}{4}, \frac{0}{4}\right), \left(\frac{1}{4}, \frac{3}{4}\right), \left(\frac{2}{4}, \frac{2}{4}\right), \left(\frac{3}{4}, \frac{1}{4}\right) \right\}$$

$$T = \{(1,1),(1,2)\}$$



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$$\mathcal{N} = \{(.00, .00), (.01, .11), (.10, .10), (.11, .01)\}$$

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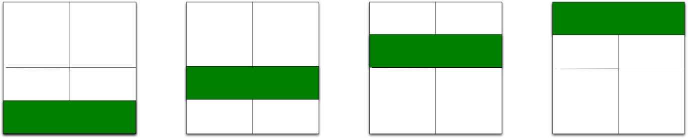

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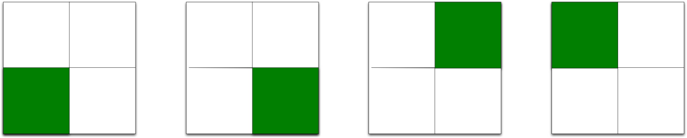
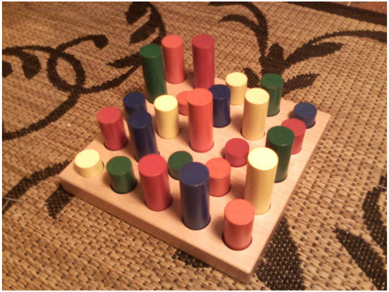
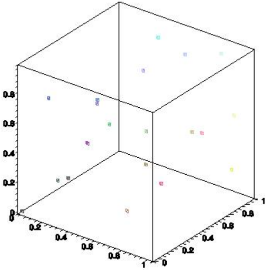


Nets from Many Sources





two mutually orthogonal latin squares of order five (color/height)

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- They show that the same works over algebraic curves (global function fields)

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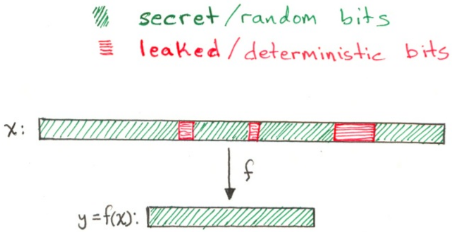
- **Research Problem:** Are there any non-trivial perfect codes in the Rosenbloom-Tsfasman metric?

Part II: Resilient Functions

Resilient Functions

How can a code be used to bolster randomness?

Resilient Functions



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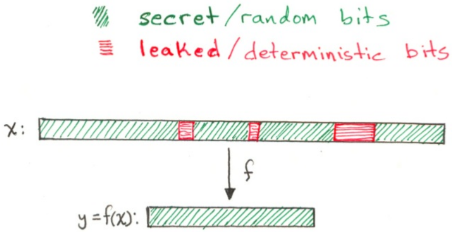

We have a secret string x. An opponent learns t bits of x, but we don't know which ones.
After applying function f, we guarantee that our opponents knows nothing.

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- **1999+:** Applications to fault-tolerant distributed computing, key distribution, quantum cryptography, etc.

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- **Why?** Any linear combination of entries of $f(x)$ is a dot product of x with some codeword
- So any non-trivial linear function of entries involves at least one random input position

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- What if quiet period is longer than expected?

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$$B_{i,r} = \sum_{\ell=0}^{k} \sum_{h=0}^{n} (-1)^{\ell-r} 2^{\binom{\ell-r}{2}} \binom{n-h}{i-h} \begin{bmatrix} \ell \\ r \end{bmatrix} A_h^{(\ell)}$$

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- E.g. $A_h^{(1)} = A_h$ for $h > 0$, $A_h^{(\ell)} = 0$ for $h < d$ except $A_0^{(0)} = 1$
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$$B_{i,r} = \sum_{\ell=0}^{k} \sum_{h=0}^{n} (-1)^{\ell-r} 2^{\binom{\ell-r}{2}} \binom{n-h}{i-h} \begin{bmatrix} \ell \\ r \end{bmatrix} A_h^{(\ell)}$$

- **Lemma** (Sunar/WJM): Let X be a random variable taking values in $\{0,1\}^n$ according to a probability distribution$\mathcal{D}_{T,A}$. Then
$$\text{Prob}[H_{\text{out}} = k - r \mid |T| = i] = B_{i,r} \binom{n}{i}^{-1}.$$

A Research Problem

Higher weight enumerators are known only for very few codes:

- MDS codes: partial information only (Dougherty, et al.)

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- Hamming codes

Can we work out these statistics for the other standard families of codes?

Part III: Fuzzy Extractors

Codes for Biometrics

How can we eliminate noise if we are not permitted to choose our codewords?

Selected References

- **1990s:** Ad-hoc mix of protocols (e.g., quantum oblivious transfer, crypto over noisy channels)

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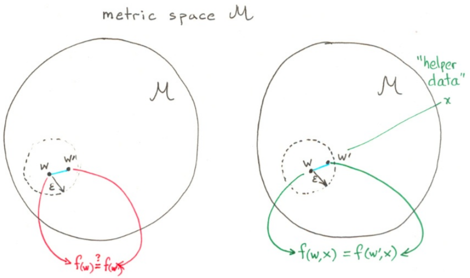
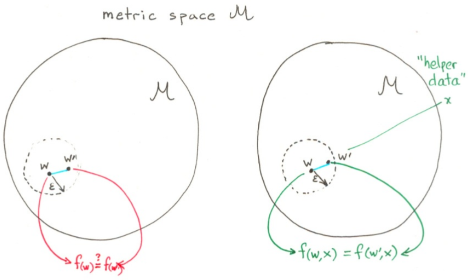
- **2009:** CD fingerprinting (Hammouri/Dana/Sunar)
- **2009:** physically unclonable functions (WPI team)

Fuzzy Extractors

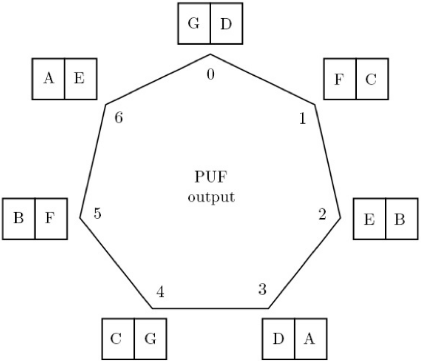


Fuzzy Extractors



Metric space $\mathcal{M}$ and function $f : \mathcal{M} \times \{0,1\}^* \to \{0,1\}^*$ such that $f(w', x) = f(w, x)$ provided x valid for w and $d(w', w) < \epsilon$.

Fuzzy Extractor: Toy Example



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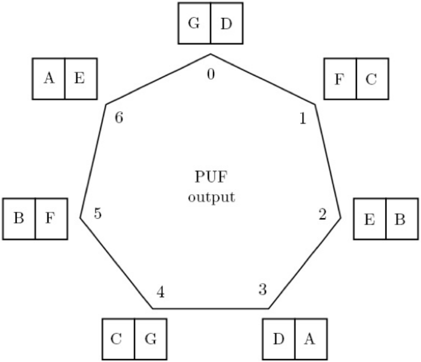

Baseline reading $w = 3$ is obtained from temporal reading $w' = 2$ and hint $x = D$.

But w is not recoverable from either w' or x alone.

Code-Offset Construction (Dodis, et al.)

Fuzzy extractor for Hamming metric:

- Start with a binary $[n, k, d]$-code with generator matrix G

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- To recover m from x and w', decode $w' + x = mG + (w - w')$
- Provided k and d are both linear in n, recovery of m from just x or w' is hard

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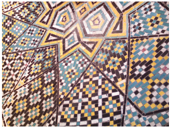
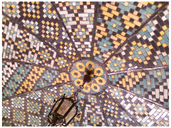
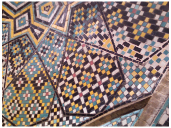
Can we build efficient fuzzy extractors for the Euclidean metric?

The End


Thank you all!