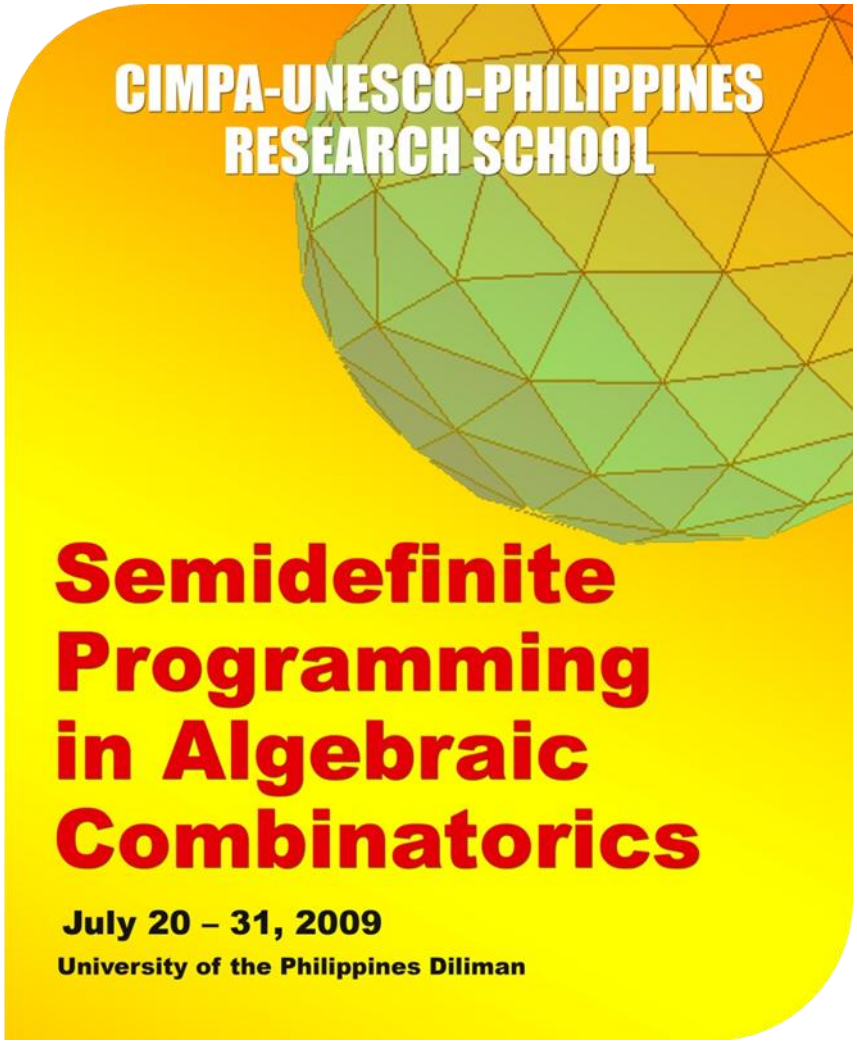

FINAL REPORT

Submitted by:

Jose Maria P. Balmaceda
Institute of Mathematics
University of the Philippines
Diliman, Quezon City

2009 CIMPA–UNESCO–PHILIPPINES RESEARCH SCHOOL ON SEMIDEFINITE PROGRAMMING IN ALGEBRAIC COMBINATORICS, Manila, Philippines
July 20 – 31, 2009

FINAL REPORT

by

Jose Maria P. Balmaceda
Local Organizer and
Director, Institute of Mathematics
University of the Philippines Diliman

Executive Summary

The CIMPA–UNESCO–PHILIPPINES Research School on Semidefinite Programming in Algebraic Combinatorics was conducted on July 20–31, 2009 at the University of the Philippines Diliman (UP).

The main organizers of the school were CIMPA and the Institute of Mathematics, University of the Philippines Diliman. The Research School was also partly supported by the Philippine Council for Advanced Science and Technology Research and Development (PCASTRD), the National Research Council of the Philippines (NRCP), the Office of the President of the University of the Philippines System, and the Embassy of France in Manila. The financial report is found in **Annex 1** of this report. CIMPA was represented by Prof. Michel Jambu, CIMPA Coordinator for Southeast Asia.

The 2009 school consisted of ten days of lectures and talks, workshop sessions, and computer sessions. Problem-solving sessions were held at the close of each lecture day. A half-day mini-conference was organized for invited talks on related topics (outside the regular school lectures). There were five lecturers (from France, USA, Japan and the Netherlands). There were 9 non-Philippine participants (from Cambodia, Indonesia, Vietnam, and India), 23 participants from Metro Manila, and 5 participants from Mindanao (in southern Philippines).

All sessions (lectures and computer sessions) were held at the Computational Sciences Research Center of the University. The facility is equipped with a computer lab and offered free internet access for all participants. All lecturers and participants from outside Metro Manila were housed at the NISMED (National Institute for Science and Mathematics Education Development) Hostel, about 200 meters from the venue of the lectures.

A website for the 2009 School (http://www.math.upd.edu.ph/cimpa_philippines_school/) was created by the Local Organizers and linked to the Institute of Mathematics homepage. Lecture notes and handouts were posted on the website, and were also distributed to all participants. A two-hour mathematical video, *Dimensione*, was provided by the CIMPA representative and was shown during lunch breaks.

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THE FULL REPORT with Annexes (Financial Report and Abstracts of Lectures)

I. Introduction

The CIMPA-UNESCO Research Schools are part of the annual scientific programme of CIMPA (International Centre for Pure and Applied Mathematics), a non-profit international organization based in Nice, France which aims to promote international cooperation in higher education and research in mathematics and related subjects, particularly computer science, for the benefit of developing countries. The Research Schools are advanced level programs consisting of around 40-60 hours of instruction and workshops that target PhD students, new PhDs and other young researchers. The University of the Philippines has hosted

CIMPA schools in 2000 (Partial Differential Equations and Related Topics) and 2005 (Pseudo Random Sequences). More recently, a CIMPA School on Numerical Methods in PDEs was organized in collaboration with the Asia-Link program IMAMIS (International Masters in Applied Mathematics and Information Science) program and was held at the Ateneo de Manila University in 2007.

The 2009 School aims to train participants in a new technique (semi-definite programming in algebraic combinatorics) which has applications in various fields such as graph theory, coding theory, association schemes, and Terwilliger algebras. Algebraic Combinatorics is a growing field of study in Southeast Asia, particularly in the Philippines. Several Filipino PhDs do research in areas such as Association Schemes, Distance-Regular Graphs, Coding Theory and Design Theory.

II. Scientific Objectives and Rationale for the School (as posted on the School Announcement)

In the seventies, Philippe Delsarte in a seminal work developed a method in Algebraic Combinatorics that yields upper bounds for the cardinality of codes with given minimal distance as a solution of a linear program. This method, also called the Delsarte method, or polynomial method, was developed in the framework of Association Schemes, which is the most general framework dealing with finite metric spaces. This method also obtains bounds in more general situations, such as lower bounds for designs (combinatorial and spherical).

To be brief, it translates problems in finite metric spaces enjoying a great degree of symmetry and/or regularity (like the Hamming scheme for codes of the Johnson schemes for designs) into spectral problems of linear algebra. These problems, in turn, can be approached by use of special functions and especially certain systems of orthogonal polynomials of one discrete variable (Krawtchouk polynomials for codes and Hahn polynomials for designs) and the extremal properties of their zeroes. The algebra of matrices that occurs in this context is the Bose-Mesner algebra.

Since the nineties, another algebra of matrices attached to association schemes has been under much study: the Terwilliger algebra. Semidefinite programs (SDP for short) constitute a special family of optimization problems which have recently become amenable to solution in polynomial time. New algorithms, based on interior point methods, have reasonable efficiency in practice. Semidefinite programs contain linear programs as a special class. More precisely, an SDP is expressed in the following way:

> Let $A_0, A_1, \ldots, A_n$ be real symmetric matrices, and $b_1, \ldots, b_n$ real numbers. One wants to find the minimum of the objective function $b_1x_1 + \ldots + b_nx_n$ over the convex set defined by the condition : $A_0 + x_1A_1 + \ldots + x_nA_n \geq 0$, where the notation $A \geq 0$ means that the eigenvalues of the symmetric matrix A are nonnegative.

SDP formulations for certain combinatorial optimization problems have been known for decades; thus the new algorithms yield polynomial time solvable relaxations for some notoriously hard problems related to graph invariants. These problems include the 1-stability number of a graph; coloring number, maximal size of a cut are also relevant examples. One of the main contributions in this area is due to L. Lovász, who introduced the so-called theta number of a graph. This number is obtained by the optimal value of an SDP, and provides an upper bound on the stability number. With this number, Lovász proved Shannon's conjecture on the capacity of the pentagon. Quite recently, by moving from LP to SDP, a number of new results and ideas have been developed in the domain of sphere packings, and these are the topics to focus on during the research school. These topics open new directions that may be fruitfully explored by young people in the future.

III. Organizers

- Christine Bachoc (University of Bordeaux I, France)
- William Martin (Worcester Polytechnic Institute, USA)
- Patrick Solé (CNRS, Nice, France)
- Local organizer:
  Jose Maria P. Balmaceda (Director, Institute of Mathematics, UP Diliman, Philippines)

IV. Lecturers and Lecture Topics

- Christine Bachoc (Bordeaux I, France) : Semidefinite programming, harmonic analysis and coding theory
- Eiichi Bannai (Kyushu University, Japan) : Review of spherical designs
- Henry Cohn (Microsoft Research, USA) : Distribution of points on spheres
- William Martin (Worcester Polytechnic Institute, USA) : Terwilliger algebras in coding theory
- Frank Vallentin (CWI, Amsterdam, Netherlands) : Applications of SDP

The lecture topics and abstracts of the lectures are in **Annex 2** of this report.

V. The Participants

A. Non-Philippine Participants (9)

	NAME	Sex	Age	COUNTRY	HIGHEST DEGREE/ SPECIALIZATION	INSTITUTION	FUNCTION
1	Ali MOHAMMADIAN	M	31	IRAN	PhD, 2007, Sharif Univ of Tech.; Algebra and Combinatorics	Institute for Studies in Theoretical Physics and Math, Tehran	Visiting Researcher
2	Pham Ngoc ANH	M	38	VIETNAM	PhD, 2007, Hanoi Inst Math; Global optimization	Dept of Scientific Fundamentals, Post and Telecom. Institute of Tech , Hanoï	Lecturer
3	Hoang Xuan VINH	M	39	VIETNAM	Master's, 2002, Hanoi Inst Math; Optimization	Hanoi Education Investment and Development (JSC), Hanoi	Lecturer
4	Vijay Kumar BHAT	M	44	INDIA	PhD, 2000, Univ of Jammu, India; algebra	School of Applied Physics and Math, Shri Mata Vaishno Devi University, Katra (Jammu and Kachemir)	Associate professor
5	Hazrul ISWADI	M	35	INDONESIA	Master's, 1999, ITB, Bandung; graph theory	Faculty of Math and Natural Sciences, Bandung Institute of Tech, Bandung	PhD student
6	Ou Phich HANG	M	29	CAMBODIA	Masters's, 2004, Royal Univ of Cambodia	Dept of Math, Shanghai Univ for Science and Tech, Shanghai	PhD student
7	Nur ROKHMAN	M	38	INDONESIA	MSc Computer Science, Gadjah Mada Univ.	Dept of Computer Science, Gadjah Mada Univ, Yogyakarta	Lecturer
8	Al SUTJIJANA	M	47	INDONESIA	MSc, 1992, Simon Fraser; algebra	Applied Math Research Group, Gadjah Mada Univ, Yogyakarta	Lecturer
9	Sutopo TOPO	M	37	INDONESIA	MSc, 2000, Gadjah Mada Univ	Applied Math Research Group, Gadjah Mada Univ, Yogyakarta	Lecturer

B. Philippine Participants (28)

B.1 Participants from Metro Manila (23)

	NAME	Sex	Age	HIGHEST DEGREE/ SPECIALIZATION	CURRENT AFFILIATION	FUNCTION
1	John Vincent S. MORALES	M	23	BS Educ (Math), 2006, DLSU	Dept of Math, De La Salle Univ, Manila	MS student
2	Michele G. TAN	F	27	MS Math, 2006, DLSU; graph theory	Dept of Math, De La Salle Univ, Manila	PhD student
3	Ederlina G. NOCON	F	41	PhD Math, 2009, Kyushu Univ, coding theory	Dept of Math, De La Salle Univ, Manila	Assoc Prof
4	Arlene A. PASCASIO	F	48	PhD Math, 1991,UPD; DRGs, finite geometry	Dept of Math, De La Salle Univ, Manila	Prof
5	Isagani B. JOS	M	46	PhD Math, DLSU; graph theory	Dept of Math, De La Salle Univ, Manila	Assoc Prof
6	Mark Anthony A. GARCIA	M	26	MS Math, DLSU; graph theory	Dept of Math, De La Salle Univ, Manila	PhD student
7	Francis Joseph H. CAMPEÑA	M	26	MS Math, DLSU; graph theory	Dept of Math, De La Salle Univ, Manila	PhD student
8	Jumela F. SARMIENTO	F	38	PhD Math, 1999, Kyushu Univ; design theory	Dept of Math, Ateneo de Manila Univ	Assoc Prof
9	Evangeline P. BAUTISTA	F	44	PhD Math, Ateneo De Manila; coding theory	Dept of Math, Ateneo de Manila Univ	Assoc Prof
10	Jayrold P. ARCEDE	M	32	MS Math; MSU-IIT	Dept of Math, Ateneo de Manila Univ	PhD student
11	Jonald P. FENECIOS	M	30	MS Math; Ateneo de Davao Univ	Dept of Math, Ateneo de Manila Univ	PhD student
12	Oreste M. ORTEGA, Jr.	M	32	MS Math	Dept of Math, Ateneo de Manila Univ	PhD student
13	Mark L. LOYOLA	M	22	BS Math	Dept of Math, Ateneo de Manila Univ	MS student
14	Abraham P. RACCA	M	24	BS Math	Dept of Math, Ateneo de Manila Univ	MS student
15	Solomon L. OLAYTA	M	28	BS Math	Dept of Math, Ateneo de Manila Univ	MS student
16	Jose Maria P. BALMACEDA	M	48	PhD Math, 1991, Urbana; group theory	Institute of Mathematics, UP Diliman	Prof
17	Ma. Nerissa M. ABARA	F	34	PhD Math, 2006, UP Diliman; matrix analysis	Institute of Mathematics, UP Diliman	Asst Prof
18	Lilibeth D. VALDEZ	F	32	PhD Math, 2006, Nice-Sophia Antipolis; coding	Institute of Mathematics, UP Diliman	Asst Prof
19	Alva Benedict C. BALBUENA	M	25	MS Math, 2009, UP Diliman; digital topology	Institute of Mathematics, UP Diliman	PhD student
20	Fidel R. NEMENZO	M	48	PhD Math, 1998, Sophia Univ, Tokyo; number th.	Institute of Mathematics, UP Diliman	Prof
21	Joseph Ray G. DAMASCO	M	20	BS Math 2009, UP Diliman; combinatorics	Institute of Mathematics, UP Diliman	MS student
22	Clarisson Rizzie P. CANLUBO	M	21	BS Math 2009; UP Los Baños; combinatorics	Institute of Mathematics, UP Diliman	MS student
23	Melvin A. VIDAR	M	39	PhD Math, 2008, DLSU; Leonard systems	Dept of Math, Univ of the East, Manila	Asst Prof

B.2 Participants from Mindanao, Southern Philippines (5)

24	Ma. Cristina L. DUYAGUIT	F	36	PhD Math, 2005, Niigata Univ, Japan; algebraic geometry	Dept of Math and Stats, Mindanao State Univ-IIT	Assoc Prof
25	Veronica B. FLORIDA	F	28	MS Math, 2005, MSU-IIT; group theory	Dept of Math and Stats, Mindanao State Univ-IIT	Asst Prof
26	Ritchie Mae T. GAMOT	F	28	MS Comp Science, 2005, UP Diliman; genetic algorithms	Dept of Math, Physics, and Computer Science, UP Mindanao	Asst Prof
27	Giovanna Fae R. OGUIS	F	24	BS Math, 2006, UP Mindanao; operations research	Dept of Math, Physics, and Computer Science, UP Mindanao	Instructor
28	Rhoda A. NAMOCO	F	32	PhD Kyoto University, 2008; optimization	Mindanao University of Science and Technology	Assistant Professor

C. There were 3 non-Philippine who were accepted but were not able to attend the School:

1	Ouaddah ABDELHAMID	M	31	ALGERIA	DES Math, 2001	Dépt de Math, Univ Djillali Liabes de Sidi Bel Abbes, Sidi Bel Abbes	Etudiant
2	Bentifour RACHID	M	28	ALGERIA	Magister, 2007; Applied Math	Dépt de Math, Univ Abou-Bekr Belkaid, Tlemcen	Etudiant
3	Rechdaoui My SOUFIANE	M	35	MOROCCO	DESA, 2002	Dépt de Math, Univ Abdelmalek Essaâdi - Tanger	Etudiant

VI. The School Programme

Generic Daily Schedule (Monday to Friday)

A.M.		P.M.	
9:30 – 10:30	Tutorial	2:00 – 3:00	Lecture 1
10:30 – 11:30	Break	3:05 – 4:05	Lecture 2
11:00 – 12:00	Tutorial activity	4:05 – 4:30	Break
12:00 – 2:00	Lunch Break	4:30 – 5:30	Group work/problem solving/discussion

Week 1 Timetable

	July 20 Mon Day 2	July 21 Tue Day 2	July 22 Wed Day 3	July 23 Thu Day 4	July 24 Fri Day 5	July 25 Sat
9:00 - 9:30	Opening program					City Tour
9:30 - 10:30	Bachoc (T1)	Martin (T1)	Bachoc (T2)	Martin (T2)	Vallentin (T1)	
10:30 - 11:00	Coffee Break					
11:00 - 12:00	Tutorial Activity					
12:00 - 2:00	Lunch Break					
2:00 - 3:00	Cohn (L1)	Cohn (L2)	Cohn (L3)	Cohn (L4)	Bachoc (L1)	
3:05 - 4:05	Bannai (L1)	Bannai (L2)	Bannai (L3)	Cohn L(5)	Martin (L1)	
4:05 - 4:30	Coffee Break					
4:30 - 5:30	Group work/problem solving/discussion					
6:30 - 8:30	Welcome dinner[3]					

Week 2 Timetable

	July 27 Mon Day 6	July 28 Tue Day 7	July 29 Wed Day 8	July 30 Thu Day 9	July 31 Fri Day 10
9:30-10:30	Mini conference[1]	Vallentin (T2)	Martin (T3)	Bachoc (L3)	Bachoc (L4)
10:30-11:00	Coffee Break				
11:00-12:00	Mini conference	Tutorial Activity[2]			
12:00-2:00	Lunch Break				
2:00-2:55	Vallentin (L1)	Vallentin (L2)	Vallentin (L3)	Vallentin (L4)	Martin (L3)
3:05-4:00	Bannai (L4)	Bachoc (L2)	Bannai (L5)	Martin (L2)	Discussions
4:00-4:30	Coffee Break				Closing
4:30-5:30	Mini conference	Group work/problem solving/discussion			
6:30 – 8:30				School Dinner	

Note:
(T) – tutorial talks (introductory talks, background material)
(L) – lecture talks (more advanced material)
[1]The mini-conference program is found on the next page of this report.
[2]There were three computer sessions as part of the tutorial activity (handled by Vallentin, Martin and Bachoc).
[3]Welcome Dinner was held at the UP Executive House, sponsored by UP President Dr. Emerlinda R. Roman.

Mini-Conference Programme
Monday, 27 July 2009, 9:30 a.m. – 5:30 p.m.

9:30 *Aspects of Coding Theory*
William J. Martin, Professor, Worcester Polytechnic Institute

10:00 *Mass Formulas for Codes over Z_p^s*
Fidel R. Nemenzo, Professor, University of the Philippines Diliman

10:30 Coffee Break

11:00 *On Zeta Polynomials of Some Codes*
Ederlina G. Nocon, Associate Professor, De La Salle University Manila

11:30 *Hermitian Orthogonality of Abelian Group Codes*
Lilibeth D. Valdez, Assistant Professor, University of the Philippines Diliman

12:00 Lunch Break

2:00 Continuation of **Frank Vallentin**'s lectures

3:00 Continuation of **Eiichi Bannai**'s lectures

4:00 Coffee Break

4:30 *Some Data on the Bilinear Form for TD Pairs of Shape (1, 2, 1)*
Melvin A. Vidar, Assistant Professor, University of the East Manila

5:00 *The Tetrahedron Algebra and the Hamming Graphs*
Arlene A. Pascasio, Professor, De La Salle University Manila

VII. Summary and Recommendations

From most indications, including feedback from the participants and lecturers, the 2009 CIMPA-UNESCO-PHILIPPINES Research School on Semidefinite Programming in Algebraic Combinatorics achieved its objectives. The lecturers were distinguished mathematicians who effectively covered the proposed lecture topics. Although there was some variance in the levels of academic preparation of the participants, the lecturers were able to provide adequate introductory material that enabled them to proceed to more advanced topics. The material covered will enable the participants to do further study in the field and pursue research work in the area.

An attractive feature of the School was the provision of daily problem solving sessions, where questions and problems. These were solved by the participants who presented their work on the board and reinforced the lessons . The problems and exercises were printed and distributed to the participants for their reference. Another valuable output of the School was the comprehensive set of Lecture Notes which the speakers prepared and distributed to all participants. Copies of important survey and research papers of the lecturers were also distributed. Three computer sessions were also conducted to introduce the participants to new software employed by researchers in the field. The Mini-Conference enabled some researchers to present related work of interest to the group. The participants were able to interact with the lecturers both inside the classroom and during meals, coffee breaks and social activities in the evening.

The Organizers were able to provide the financial requirements to conduct the School. The funding (split 2/3 - 1/3 between CIMPA/France and the Philippines) supported the travel, board and lodging of the speakers, and participants. Although there was initial concern about the financial requirements for the school, in the end, there was enough financial support solicited. No registration or other fees were charged. The financial assistance provided the opportunity for well-motivated and qualified participants from Southeast Asia to benefit from the activity.

The months of July and August are popular months for holding a typical Research School. However, as the school calendar in the Philippines is different (first term starts in June and ends in October), there were some potential participants who were not able to attend the School because of conflicts with their graduate classes or their teaching duties. Nevertheless, there were 28 official Philippine participants who attended fully. This included 5 young mathematicians from the island of Mindanao in Southern Philippines. In addition, some faculty members of the Institute of Math of UP Diliman were able to attend partly the sessions.

Effort should be made to attract and accommodate participants from other countries in the region, specifically Cambodia, Laos, Thailand, and also Bangladesh and Pakistan. To enable those who may not have enough background for the more advanced topics, perhaps a one-week preparatory program may be organized. Local mathematicians may handle the introductory and preparatory lectures.

In closing, the local organizers wish to thank CIMPA, particularly Professors Claude Cibils and Michel Jambu, for their generous support for the Research School. The organizers also thank the Embassy of France in Manila (represented by scientific attaché Mme. Isabelle Epaillard, who graced the occasion) for its support and assistance. The participants and the local organizers commend the Lecturers for providing their valuable time and effort and their kindness and friendship. Finally, the full support of the Institute of Mathematics, the Office of the UP President, the Philippine Council for Advanced Science and Technology Research and Development, and the National Research Council of the Philippines is gratefully acknowledged.

The UP Diliman Institute of Mathematics and the local Philippine mathematical community look forward to participation in future Research Schools and more collaboration with CIMPA.

END OF REPORT

ANNEXES:

1. FINANCIAL REPORT
2. LECTURE TOPICS AND ABSTRACTS OF LECTURES
3. PHOTOS

ANNEX 1

FINANCIAL REPORT
CIMPA-UNESCO-PHILIPPINES 2009: Semidefinite Programming in Algebraic Combinatorics

July 20 – 31, 2009
University of the Philippines Diliman

	ITEM	TOTAL (€)	FRANCE		PHILIPPINES			
			CIMPA	French Embassy Manila	UP MATH	PCAS-TRD	UP-OP	NRCP
I.	**Accommodation expenses** • Speakers (5); Local Organizer (1) • CIMPA representative (1) • Non-Philippine participants (9) • Philippine participants from outside Metro Manila (5)	**1,855**	1,855					
II.	**Allowances for breakfast and dinner**							
	• Speakers and non-Philippine (15)	**1,575**	1,575					
	• Philippine participants from outside Metro Manila (5 persons)	**300**			300			
III.	**Food expenses during the school**							
	• Lunch, snacks, coffee at €6.8/day x 50 participants and staff x 10 days	**3,400**	1,020		1,280	1,100		
	• Opening reception	**400**					400	
	• Closing reception	**400**			400			
IV.	**Local transportation and tour**	**500**			500			
V.	**Organizational Expenses** • Supplies, facilities and room rentals, communication, technical assistance, printing of course notes, meals and snacks of local organizing committee	**1,500**			1,200			300
VI.	**Air fare and other support**							
	• Speakers and non-Philippine	**7,500**[1]	7,000[1]	500				
	• Philippine participants from outside Metro Manila (5)	**400**				400		
	TOTAL EXPENSES	**€ 17,830**	11,450	500	3,680	1,500	400	300
	COST-SHARING	**100%**	64.2%	2.8%	33%			

Acronyms and Notes:
UP MATH – Institute of Mathematics, University of the Philippines Diliman
UP-OP – Office of the President, University of the Philippines System
PCASTRD – Philippine Council for Advanced Science and Technology Research and Development
NRCP – National Research Council of the Philippines
[1]Approximate amount (exact amount to be supplied by CIMPA)

Submitted by:

Jose Maria P. Balmaceda
Director, Institute of Mathematics and Local Organizer
Email: jose.balmaceda@gmail.com, director@math.upd.edu.ph
Tel/Fax: +632-920-1009; Tel: +632-928-0439

ANNEX 2

CIMPA-UNESCO-PHILIPPINES 2009 RESEARCH SCHOOL
LECTURE TOPICS AND ABSTRACTS OF LECTURES

CHRISTINE BACHOC, Bordeaux I, France
http://www.ufr-mi.u-bordeaux.fr/~bachoc/

Semidefinite programming, harmonic analysis and coding theory

(1) Introduction to coding theory
(2) Introduction to group representations
(3) A general framework for LP/SDP bounds using group representations
(4) Applications to grassmanians, unitary groups, list decoding, etc.
(5) Optimization problems from group representations
(6) Recap of algebraic theme of the research school

We will try to present a general framework based on representation theory that obtains upper bounds for sphere packing problems from semidefinite programming. We will first review on the classical cases of two-point homogeneous spaces dealt with Delsarte linear programming method, with emphasis on the connections with the orthogonal polynomials in one variable. We shall recall the most beautiful cases were some explicit polynomials give exact bounds and proofs of optimality and uniqueness of some codes.

Then we will show how to extend this method to more general spaces, introducing semidefinite programming bounds and orthogonal polynomials with several variables. We will review many cases, where this method allows either to strengthen the Delsarte method, or to treat spaces that could not be dealt with before.

EIICHI BANNAI, Kyushu University, Japan
http://hyoka.ofc.kyushu-u.ac.jp/search/details/K000379/english.html

Review of spherical designs

(1) Spherical codes and designs
(2) Algebraic combinatorics on spheres I
(3) Algebraic combinatorics on spheres II
(4) Toy models for D.H. Lehmer's conjecture
(5) Euclidean designs and coherent configurations

Spherical designs are an analytic generalization of combinatorial designs. They bear connections to numerical integration, euclidean lattices, and modular forms. The course will survey both construction and bounds on the size.

HENRY COHN, Microsoft Research, USA
http://research.microsoft.com/~cohn/

Distribution of points on spheres

(1) Introduction to spherical harmonics
(2) Introduction to LP and SDP on spheres
(3) Bounds for codes and packings
(4) Energy minimization on spheres and applications I
(5) Energy minimization on spheres and applications II

One natural generalization of sphere packing is the energy mimimization problem: given some potential function depending on the pairwise distances between points, how should the points be arranged so

as to minimize the total energy? This problem arises naturally in geometry, information theory and physics. Abhinav Kumar and I have introduced the idea of a universally optimal configuration (for example, all inverse power laws). This gives a natural notion of the best way to distribute points on a surface such as a sphere or in space. The first class studies universal optimality theoretically, the second analyzes a surprising example in detail, and the third collects numerical evidence from massive computer searches and examines many interesting point configurations.

WILLIAM MARTIN, Worcester Polytechnic Institute, USA
http://users.wpi.edu/~martin/

Terwilliger algebras in coding theory

(1) The Hamming graph and the linear programming bound
(2) Specific solutions to linear programming problems
(3) The Terwilliger algebra of the n-cube and its positive semidefinite cone
(4) Linear programming techniques for the n-cube
(5) Association schemes and its applications
(6) Recap of the optimization theme of the research school

In the seventies Philippe Delsarte in a seminal work developed a method initially aimed at bounding codes over finite fields, that yields upper bounds for the cardinality of codes with given minimal distance as a solution of a linear program. This method, also called the Delsarte method, or polynomial methods, was developed in the framework of Association Schemes, which is the most general framework dealing with finite metric spaces and was also linked to the Lovasz theta number by A. Schrijver. This method also obtains bounds in more general situations, such as lower bounds for the size of combinatorial designs.

FRANK VALLENTIN, CWI, Amsterdam, the Netherlands
http://homepages.cwi.nl/~vallenti_n/

Applications of semidefinite programming

(1) Introduction to SDP I
(2) Embedding problems for metric spaces
(3) Introduction to SDP II
(4) Bourgain's Theorem and applications
(5) MAXCUT algorithms

Semidefinite programming can be applied to a variety of problems in geometric and algebraic combinatorics. We will present three examples in detail: constructing sphere coverings, finding upper bounds for packing problems, and computing low distortion embeddings of finite metric spaces.

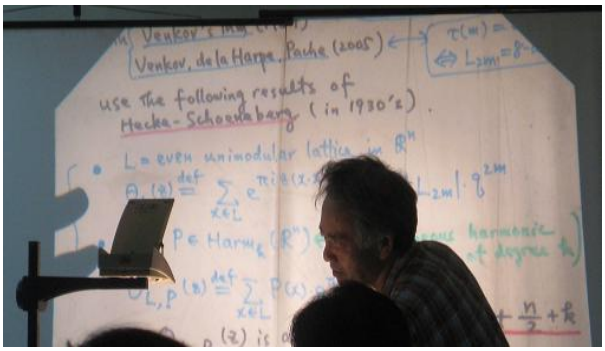
The sphere covering problem is a classical problem in the geometry of numbers. Roughly speaking, it is concerned with minimizing the number of unit spheres which are need to cover arbitrary large but finite regions of n-dimensional Euclidean space. If one restricts the problem to the case when the centers of the spheres form a periodic point set one can solve the problem computationally using a mixture of polyhedral combinatorics and semidefinite programming.

One can formulate many problems in combinatorial optimization and geometry as a packing problem in an underlying metric space. Examples include the kissing number problem of the stability number of a graph. We will explain a generic method based on semidefinite programming and harmonic analysis to find upper bounds for packing in compact metric spaces.

The theory of low distortion embeddings of finite metric spaces has found many applications in theoretical computer science in the last years. Especially embeddings of finite graphs where the metric is given by the shortest path have been used to design approximation algorithms. The determination of an embedding with lowest distortion can be done by semidefinite programming. If the graph has structure, for instance symmetry or high girth, one can apply techniques from algebraic combinatorics and orthogonal polynomials to prove some analytic properties of the optimal embedding.

ANNEX 3

SCENES FROM THE 2009 SCHOOL IN MANILA, PHILIPPINES



Clockwise from top: (1) Group photo; (2) Prof Eiichi Bannai; (3) aerial view of venue (building in middle of picture inside oval) with the Institute of Mathematics to its left and NISMED hostel at northeast corner; (4) Lecturers with students at the UP Los Baños campus; (5) Prof William Martin