Discrete Math Day
Problem Session Contribution
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Equiangular Lines in Complex Space

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Problem: Find 306 lines in $\mathbb{C}^{18}$ such that any two form the same angle. Equivalently, for $d = 18$, find $d^2 - d$ unit vectors $\{\mathbf{v}_i \mid 1 \leq i \leq d(d-1)\}$ such that, for some real α, $|\mathbf{v}_i \mathbf{v}_j^*| = \alpha$ for all $i \neq j$.

Discussion: A set of d^2 equiangular lines in $\mathbb{C}^d$ is called a "symmetric informationally complete positive operator-valued measure" (SIC-POVM, for short) and these are of interest in quantum information theory, e.g., for quantum state tomography. In order to attain d^2 vectors, it is easy to prove that $\alpha = 1/\sqrt{d+1}$ is required.

Gerhard Zauner conjectured in 1999 that these exist for all d and conjectured a method for constructing them. Exact solutions have been found by Gröbner basis methods for $2 \leq d \leq 17$, and $d = 19, 24, 35, 48$. High-precision numerical approximations have been found for $d \leq 121$. So $d = 18$ is the smallest open case for an exact solution. Here, we instead ask for $d^2 - d$ equiangular lines in the hopes that (a) the problem may be more tractible and (b) a solution to the easier problem may provide insight into the SIC-POVM problem.

References: A survey article can be found here: `http://arxiv.org/abs/0910.5784` and a recent talk which provides an update can be found here:
`http://users.wpi.edu/ martin/MEETINGS/LINESTALKS/Appleby.pdf`

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