LECTURE 4
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Objectives

• What is the difference between a lumped and distributed system?

• What are the characteristic parameters of a Transmission Line?

• What sets the voltages and currents apart from low frequency lines?
General transmission line equations

• Detailed analysis of a differential section

Note: Analysis applies to all types of transmission lines such as coax cable, two-wire, microstrip, etc.
Kirchhoff’s laws on a microscopic level

- Over a differential section we can again use basic circuit theory
- Model takes into account line losses and dielectric losses
- Ideal line involves only L and C
Advantages versus disadvantages of electric circuit representation

- Clear intuitive physical picture
- Yields a standardized two-port network representation
- Serves as building blocks to go from microscopic to macroscopic forms
- Is a one-dimensional representation (does not take into account interferences from neighboring elements)
- Material nonlinearities, (hysteresis) and temperature effects are not included
Derivation of differential transmission line form

**KVL:**

\[
V(z) = (R + j\omega L)I(z)\Delta z + V(z + \Delta z)
\]

\[
-Lim\left(\frac{V(z + \Delta z) - V(z)}{\Delta z}\right) = -\frac{dV(z)}{dz} = (R + j\omega L)I(z)
\]

**KCL:**

\[
I(z) = (G + j\omega C)\Delta z V(z + \Delta z) + I(z + \Delta z)
\]

\[
-\frac{dI(z)}{dz} = (G + j\omega C)V(z)
\]
Traveling voltage and current waves

\[
\frac{d^2 V(z)}{dz^2} - \gamma^2 V(z) = 0 \quad \text{Left traveling wave}
\]
\[
\frac{d^2 I(z)}{dz^2} - \gamma^2 I(z) = 0 \quad \text{Right traveling wave}
\]

where

\[
\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}
\]

Phasor expressions

\[
V(z) = V^+ e^{-\gamma z} + V^- e^{+\gamma z}
\]

\[
I(z) = I^+ e^{-\gamma z} + I^- e^{+\gamma z}
\]
Characteristic impedance

\[ I(z) = \frac{\gamma}{(R + j\omega L)} (V^+ e^{-\gamma z} - V^- e^{+\gamma z}) \]

\[ Z_0 = \sqrt{\frac{(R + j\omega L)}{(G + j\omega C)}} = \frac{V^+}{I^+} = -\frac{V^-}{I^-} \]

Characteristic line impedance