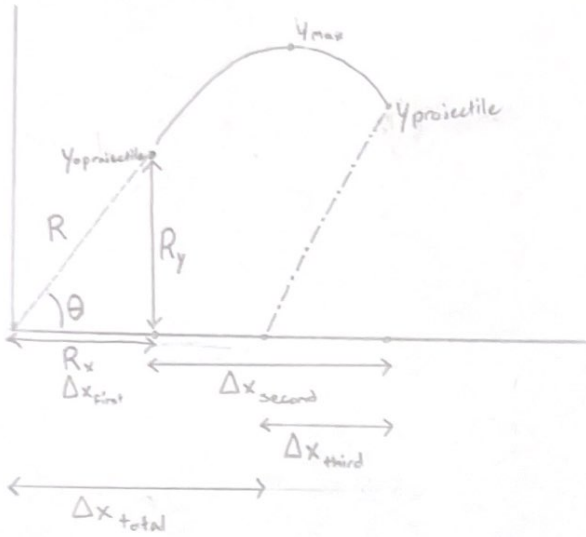

Lydia Metcalf

Multi-Stage Rocket Problem

09/24/24

First Step

We first find the distance along the line that the rocket travels. We know that $t_{first} = 8.1\,s$, $a_{first} = 6.4\ m/s^2$, and $v_0 = 0\ m/s$, as the rocket was launched from rest. We use the no-v equation to solve for d (modified slightly to avoid using x or y as variables).

$$d = v_0 + \frac{1}{2}at^2$$
$$d = 0 + \frac{1}{2}(6.4)(8.1)^2$$
$$d = 209.952\ m$$

From here, we have a vector problem. We know that $R = d = 209.952\ m$ and $\theta = 51°$. We then solve for the vector components and take $R_x = \Delta x_{first}$ and $R_y = \Delta y_{first}$. Since $y_0 = 0\,m$, we have $y_{first} = \Delta y_{first}$ as well.

$$R_x = R \cos \theta°$$
$$R_x = 209.952 \cos 51°$$
$$R_x = 132.127\ m$$
$$\Delta x = 132.127\ m$$

$$R_y = R \sin \theta°$$
$$R_y = 209.952 \sin 51°$$
$$R_y = 163.163\ m$$
$$\Delta y = 163.163\ m$$
$$y_{first} = 163.163\ m$$

Finally, we solve for v_{first}, the velocity at the end of the first stage. Since we have $t_{first} = 8.1\ s$, $a_{first} = 6.4\ m/s^2$, and $v_0 = 0\ m/s$, we use the no-x equation to find v_{first}.

$$v = v_0 + at$$
$$v = 0 + 6.4(8.1)$$
$$v_{first} = 51.84\ m/s$$

Second Step

First, we solve for y_{max} given that $\theta = 51°$, $v_0 = 51.84$ m/s and $y_{o\,projectile} = y_{first} = 163.163$ m, and using $a = -9.8$ m/s². We use the no-t equation, plugging in $v_0 \sin\theta°$ for v_{oy}, and using $v_y = 0$ to get the Δy from $y_{o\,projectile}$ to y_{max}. Then, we add Δy_{second} to $y_{o\,projectile}$ to get y_{max}.

$$V_y^2 = V_{oy}^2 + 2a\Delta y$$
$$0^2 = (51.84 \sin 51°)^2 + 2(-9.8)\Delta y$$
$$0 = (40.287)^2 - 19.6\,\Delta y$$
$$\Delta y_{second} = \frac{(40.287)^2}{19.6}$$
$$\Delta y_{second} = 82.808 \text{ m}$$
$$y_{max} = y_{o\,projectile} + \Delta y_{second}$$
$$= 163.163 + 82.808$$
$$= 245.971 \text{ m}$$

From y_{max}, the rocket falls 60 m vertically before the parachute opens. Then, we can find $y_{projectile}$, the height at the end of the second stage.

$$y_{projectile} = y_{max} - 60$$
$$= 245.971 - 60$$
$$= 185.971 \text{ m}$$

Now that we have $y_{o\,projectile}$ and $y_{projectile}$, we can solve for Δx_{second} as with any projectile.

Horizontal	Vertical
$\Delta x_{second} = V_x \cdot t_{second}$	$y_{projectile} = y_{o\,projectile} + V_{oy} t_{second} + \frac{1}{2} a t_{second}^2$
$\Delta x_{second} = 51.84 \cos 51°\, t_{second}$	$185.971 = 163.163 + 51.84 \sin 51° t - 4.9t^2$
$\Delta x_{second} = 32.624\, t_{second}$	$0 = -4.9t^2 + 40.287t - 22.808$
$\Delta x_{second} = 32.624(7.6102)$	$t_{second} = 7.6102$ s
$\Delta x_{second} = 248.275$ m	(plug into horizontal side)

Third Step

Because the horizontal and vertical speeds are independent of each other, we can calculate the time it will take to fall to the ground. We're given that $v_{y\,parachute} = -6.0$ m/s, $y_{o\,parachute} = y_{projectile} = 185.971$ m, and constant velocity. Because $y_{parachute} = 0$ m (ground), $\Delta y = -y_{o\,parachute} = -185.971$ m. Then, we solve for Δx_{third}, given $t_{third} = 30.995$ s and $v_x = -15$ m/s.

$$t_{third} = \Delta y_{parachute} / V_{y\,parachute}$$
$$t_{third} = -185.971 / -6.0$$
$$t_{third} = 30.995 \text{ s}$$
$$\Delta x_{third} = V_x(t_{third})$$
$$\Delta x_{third} = -15(30.995)$$
$$\Delta x_{third} = -464.925$$

Adding the displacement for each step, we have $\Delta x_{total} = \Delta x_{first} + \Delta x_{second} + \Delta x_{third}$

$$= 132.127 + 248.275 - 464.925$$
$$= -84.523 \text{ m}$$

So the displacement is $\boxed{-84.52 \text{ m}}$.