- **Question:** Does the relationship between force, mass, and acceleration of a cart traveling along a metal track in a modified Atwood's machine on an inclined plane, as seen in figure 1, obey Newton's Second Law?
- **Hypothesis:** As the angle of the inclined plane of the modified Atwood's machine increases, the acceleration up the ramp decreases. The relationship between acceleration and the sin of θ would be linear. The slope of the graph of acceleration vs. sin θ would be equal to the acceleration due to gravity multiplied by the mass of the dragging cart all over the combined masses of the carts. The y-intercept would also represent the acceleration if the angle was 0.

Strategy:

- The hanging mass was kept equal to the mass on the cart. The resulting acceleration was measured using a Vernier motion detector.
- The angle between the surface and the track was changed using textbooks placed under one end of the track and the angle was measured with a protractor. The angles were kept small enough such that M₁ would not move down the ramp.
- The acceleration was graphed vs. the sin of the angle to verify that the slope was equal to the acceleration due to gravity multiplied by the mass of the dragging cart all over the combined masses of the carts.

Data:

Masses of each cart = 0.3024 kg

| Angle | sin(Angle) | Average Acceleration |
|-------|-------------|----------------------|
| 0 | 0 | 4.755 |
| 6.5 | 0.113203214 | 4.045 |
| 15.3 | 0.26387305 | 3.592 |
| 22 | 0.37460659 | 3.019 |

Analysis:

The free body diagrams in Figure 2 show the forces on the masses in the modified Atwood's machine.



Figure 2: Free Body Diagrams

Friction

between the cart and the track is negligible because the cart's wheels spin freely. The following equations are based on the free body diagrams. Positive motion is defined as to the right for the cart, and down for the hanging mass.

$$T = m_1 gsin(\theta) + m_1 a$$

$$F_{G_2} - T = m_2 a$$

These equations can be combined to form the equation

$$a = \frac{-m_1 g}{m_1 + m_2} sin(\theta) + \frac{m_2 g}{m_1 + m_2}$$

This equation indicates that there is a linear relationship between the sin of the angle of the incline and the acceleration. The slope of this line should be the half of the acceleration due to gravity, as $m_1 = m_2$.

A graph of acceleration vs. $\sin(\theta)$ data for this experiment shows that it is indeed linear, and that the slope is equal to -4.41, which is around half of the acceleration due to gravity. It also shows that the acceleration, if the angle was 0, would be around 4.68 m/s.



Figure 3: Acceleration vs. $sin(\theta)$

The actual half of the acceleration due to gravity is 4.9 m/s², which means that there was a 10% error in our data. The fact that it is too small indicates that the most likely source for this discrepancy is friction in the wheels of the cart or air resistance in the system.



Figure 1. Modified Atwood's Machine on an

Inclined Plane