

WPI Physics Dept.  
Intermediate Lab 2651  
Magnetic Field Mapping

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# 1 Background Material and Experimental Procedure

## 1.1 Introduction.

Magnetic Fields are obtained in the lab in one of two ways. A permanent magnet is a piece of material, such as iron, in which the magnetic dipole moments of the constituent atoms are aligned. The resulting B field is maintained without the need for any outside energy source, as long as the atoms stay aligned. The second way of creating a B field is with a current-carrying wire, in which case the B field disappears as soon as the current stops. In this lab you will use a Hall effect probe to study the magnetic field distribution around both permanent magnets and current carrying wires. Current geometries to be studied include a straight wire, a single loop, and a so-called Helmholtz coil pair. In preparation for this lab you should review the calculation of magnetic field for these geometries using the Bio-Savart law and review the physical principles behind the Hall effect (both topics should be covered in the books you used in your 1st year physics courses.)

## 1.2 Preparations.

To get familiar with the Hall effect probes, you can begin by measuring the magnetic field of a small permanent magnet. With assistance from the instructor, identify the various features of the control panel of both the Lakeshore Gaussmeter. Check out how to adjust the sensitivity range, how to use the zero-suppress or “relative” key, and how to adjust the coarse and fine zero. For our preliminary measurements, set the sensitivity on the Gaussmeter to kG scale (note that  $1\text{G}=10^{-4}\text{T}$ ).

\*\*\*\*\*CAUTION\*\*\*\*\*

The Hall probes are delicate—handle with care. Be careful not to bend it.  
\*\*\*\*\*

The small magnet is in the form of a cylinder, with a dipole magnetic field coming out one end of the cylinder and back into the other end. Holding the magnet in your hand, move it around in the vicinity of the probe and note the positions where you get a strong reading. If the reading starts to go negative, then either reverse the orientation of the magnet or flip the

polarity on the meter. Find the approximate position on the probe which gives a maximum reading. This is where the Hall effect element (about  $1\text{mm}^2$  in area) is located. In analyzing your data in this lab, you will need to take into account the distance from the tip of the probe to this active area. Measure this distance as well as you can by placing the magnet on a nonmagnetic support so as to maintain a separation of about  $1\text{mm}$  between the flat face of the probe and the flat end of the magnet. Then slide the magnet along the length of the probe and, when the reading is maximized, measure the distance from the center of the magnet to the probe tip.

### 1.2.1 Large Permanent Magnet

Set the scale of the Gaussmeter at  $1\text{kG}$  and put the large permanent magnet on the table with the pole faces pointing upwards. Arrange the probe so that it is near the center of the flat region between the two poles of the magnet, and rotate the probe so as to obtain a maximum reading. How does the strength of the large permanent magnet compare to that of the small one? The reading changes as the probe is rotated because the Hall voltage depends only on the component of B field which is perpendicular to the plane of the probe. Any component parallel to this plane does not produce a Hall voltage. (Make sure you understand why this is true—look in your books to review the Hall effect). Note the resulting orientation of the probe tip, and ask yourself if that makes sense. Move the probe around between the pole faces to test for uniformity. Determine approximately how much the B field varies in the region between the pole faces (i.e. what is the fractional change?)

Now position the probe close to the center of the region between the pole faces. Rotate the probe through  $180^\circ$ , taking readings every  $5^\circ$  or  $10^\circ$  as appropriate. Since the Hall probe responds only to the component of the field perpendicular to the plane of the probe tip, the variation of reading with angle is expected to be sinusoidal (why?). Do a quick check in the lab to verify this quantitatively, by comparing the peak reading to the reading  $45^\circ$  off peak. In your report, do not simply plot B vs. angle. Rather, you should plot your data in such a way that you expect theoretically to get a straight line.

### 1.2.2 Straight Wire

Another fundamental field is that generated by a straight wire, whose magnetic field you have determined in the Pre-Lab exercise. Here you will use the big square wire loop attached to the wall. Note this loop actually carries 5 conductors, connected to a breakout panel with a switch. If current passes through all five loops, then the magnetic field is five times as strong as that passing through just one loop. Measure the azimuthal magnetic field as a function of the radial distance of the probe - be sure the axial probe is aligned properly such that the B-field passes through the active square of the Hall probe. Do this for one single conductor and all five conductors. Note the wires are  $\sim 18$  gauge and can safely take up to  $\sim 16$  Amps of current. However the ammeter can only handle smaller currents - too much current will blow its fuse!

In order to check for any background signal, decrease the current to zero (keep the current knob fixed, and turn down the voltage knob). Depress the “relative” key on the Gaussmeter to achieve a zero reading. Now slowly move away from the wire and note the change in the B field reading. Can you explain why the magnetic field would vary with position when the current is off? In order to measure the B field due to the wire, this “background” reading with the current off must be subtracted from the reading taken with the current on for every value of  $z$  for which we make a measurement. The background subtraction becomes more important when the signal is small—i.e., far away from the wire.

Fortunately, we can readily subtract the background field using the “relative” key on the Gaussmeter. For each position  $z$ , turn the current off using the voltage knob, press “relative”, turn the current back up to 10.0 A, and record the reading. The reading is the contribution to the total B field due to the current in the wire.

Since we are interested in how the B field strength varies as a function of the distance away from the wire, you will want to choose smaller steps in positions close to the wire, and larger steps farther away. In deciding on the appropriate step size, consider how much the reading changes between steps. This change should not be greater than 20% or so in any given step. If you follow this 20% rule, you will have enough points to plot a good graph. Continue taking reading up to 50 cm away from the wire. For a long straight wire, the B field is expected to fall off as  $1/r$ , where  $r$  is the distance from the center of the wire. Make a plot of the corrected B vs.  $1/r$ , and see

if comes out a straight line. Can you explain any observed deviation from the expected behavior? Be sure in your analysis that you properly correct for the position of the active region of the probe relative to the tip, and for the thickness of the wire. Make clear in your report how you are obtaining corrected values for  $B$  and  $r$ .

When finished with this part of the experiment, turn down both the voltage and the current control knobs all the way (counterclockwise).

### 1.2.3 Single Coil

For the single coil and Helmholtz coil measurements, we are interested in the field along the axis of the coil. The location of the active region in the longitudinal Hall probe may be found using the fact that the  $B$  field along the axis of a single coil has a maximum right at the center of the coil.

Connect one of the coils in series with the power supply and an ammeter. Measure the diameter of the coil, and note the number of turns, which is written on the coil. To keep from overheating the coil, the current should always be kept below 2 A, so 1 A is reasonable.

Measure the variation in  $B$  field along the symmetry axis of the coil, from as close as possible out to 30 cm. As before, use the switch to reverse the current through the Helmholtz coil, and remove the background fields. Also use the 20% rule to determine the step size.

When finished with this part of the experiment, turn the current down to zero.

### 1.2.4 Helmholtz Coil

Connect the second coil in series with the first coil, in such a way that the magnetic fields add to give a larger field in between the two coils. The coils are in the Helmholtz configuration when the separation  $L$  between the coil centers is equal to the radius  $R$  of each coil. To begin, adjust the separation to be a centimeter or two larger than the expected Helmholtz value. Before turning on the current again, have the instructor check the connections. Increase the current to 400 mA, as measured by the series ammeter. The  $B$  field is expected theoretically to vary with axial position  $z$  as shown in Fig.1 below. Near the Helmholtz condition, the variation of  $B$  with  $z$  will be very small, so it is easier to look for the variation in  $B$  if you exploit the “relative key” on

the Gaussmeter to subtract most of the large non-zero field near the center of the two coils.

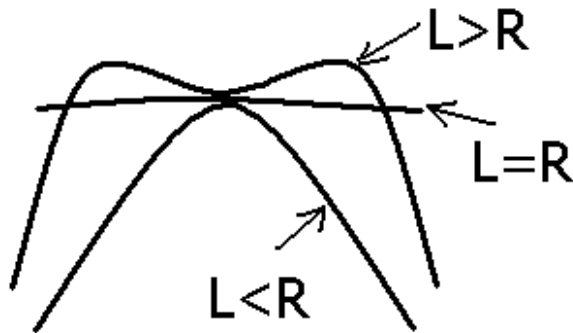


Figure 1: Variation of B field with position when the distance  $L$  between the coils is close to the radius  $R$  of the coils.

As shown in Fig. 1, the field distribution is double-peaked when  $L > R$ , but it is single-peaked for  $L < R$ . After pressing the “relative key” move the probe along the symmetry axis  $z$  and note whether the variation with  $z$  is single-peaked or double-peaked. If it is double-peaked, move the coils a little closer together and test it again. If it is single-peaked, move the coils a little farther apart. Keep repeating this, until the variation with  $z$  is just at the boundary between single and double-peaked, which is the Helmholtz condition  $L = R$ . When you are satisfied with this, record the position of each coil on the rail, and measure the coil separation with a ruler.

With the coils now in the Helmholtz configuration, measure the B field strength vs. position  $z$  along the symmetry axis, for points both between and outside the coils, up to 30 cm away. Between the coils, take an interval of 0.5 cm between points; outside, choose an appropriate interval which will give you a good curve to draw. In your report, you should derive a formula for the dependence of field on position in this configuration, using your previous result for a single coil. For your analysis, plot three curves in the same figure: experimental curve, theoretical curve, and the ratio between them. Plotting the ratio  $B_{exp}/B_{theory}$  vs. position is useful because it allows you to see if the data agrees with theory to within a scaling factor. Since there is a very small fractional variation between the coils it would be best to make a separate graph of this region, with an appropriate scale.

## 2 Pre-lab Exercises

1. Calculate  $B$  ( both magnitude and direction ) at a 1.0 mm distance from an infinitely long straight wire which is carrying a current of 10.0 A. Express answer both in units of tesla (T) and gauss (G). Repeat for distances of 1.0 cm, 10 cm, and 50 cm.
2. The Earth's magnetic field has a strength of about 0.5 gauss in Olin Hall. At what distance from a long straight wire carrying 10.0 A is the field caused by the current equal to the earth's field?
3. Consider the magnetic field created by a current flowing in a long straight wire at a point 50 cm from the wire. By how much is the field changed if the straight wire portion contributing to the field is only 1.00 m long instead of infinitely long? Does it matter where the 1 m segment is located? Explain.
4. Calculate  $B$  (both magnitude and direction) at the center of a diameter  $D = 16$  cm coil containing 320 turns and carrying a current of 400 mA.
5. The Earth magnetic field has a strength of about 0.5 gauss in Olin Hall. At what distance from the center of the above coil and along its symmetry axis will the field due to the current be equal to 0.5 gauss?
6. Derive the expression for the magnetic field produced by a finite-length straight wire at a distance  $r$  from the line of the wire. Start with Eq. (2) and derive Eq. (3).

### 3 Experimental Checklist

For the first day:

1. Determine the location of the sensitive region of the newer transverse Hall probe used in the long straight wire experiment:  $d \pm \delta d$ .
2. Track the behavior of the Hall probe (using the older of the two Gaussmeters) between the poles of the permanent magnet.
3. Be sure to adjust the Gaussmeter zero for equal positive/negative maximum values.
4. Measure every  $10^\circ$ , 21 measurements, over a  $200^\circ$ -range that includes two peaks values.
5. Long straight wire experiment:
  - Measure the parameters of the long straight wire (diameter; length).
  - Check the field prediction quickly (with the corresponding Get-Acquainted Exercise calculation) so that you will have something to compare your experimental result with.
  - Adjust the zero to null out the background with the transverse probe adjacent to the wire. Set the current to 10.0 A and measure the corresponding field. Compare with prediction.
  - Measure the field at several points from a point adjacent to the wire to a point 50 cm from the wire; increase  $r$  enough each time to decrease field by about 50%; take about 9 measurements total; for the last several readings when the measured field is less than 100 times the background reading, zero out the background reading of the Gaussmeter before making a measurement of the field associated with the 10-Amp current. Also, repeat a couple of measurements to see how repeatable your measurements are. (This will give you a measure of the precision of your B-measurements.)

For the second day:

1. Single coil experiment:



- Measure and observe the parameters of the coil, such as the inside and outside diameter of the coil, its thickness, and the number of wire turns per coil.
- Align the axial Hall probe with the center of coil and align the two sets of rails to be parallel.
- Adjust the coil current supply to an output of 500 mA; turn off the current in order to null out the background reading on the Gaussmeter; spot-check the null over the full z-range.
- The sensitive spot in the axial Hall probe is approximately 1 mm in from the tip of the probe. Figure out a procedure that will allow you to check to see if this is indeed the case.
- Measure the field from a point close to the center of the coil ( $z = 0$ ) out to 30 cm distance; increase  $z$  enough each time to decrease the field to about 60% of the previous value. Take about 9 measurements total; for the last several readings when the measured field is less than 100 times the background reading, zero out the background reading of the Gaussmeter before making a measurement of the field associated with the 500 mA current. As with the straight wire, repeat a couple of measurements to see how repeatable your measurements are.
- Check results quickly with the corresponding Get-Acquainted Exercise calculation to see if they make sense.

## 2. Helmholtz pair experiment:

- Connect the second coil into the circuit and adjust the current to 500 mA. Place the coils at a spacing of  $L \approx R$ ; slide the axial Hall probe to a  $z$ -value in between the coils; follow the technique described in the instructions for setting  $L = R$ .
- Measure the field at five points, starting at the inside edge of the left-hand coil, then going to the mid-point between coils, then 1.0 cm to either side of center, and finally ending at the inside edge of the right-hand coil. Measure the background readings at the same points.
- Move the Hall probe to the center of the right-hand coil and repeat measurements as in 4e; correct for the effect of the background field as in 4e above. Spot-check repeatability as above.

## 4 Report Checklist

1. Hall probe measurement as a function of angle In addition to providing a table containing your data and derived quantities, you are to graph the data in such a way so as to expect a straight-line relationship. One possibility is  $B_{\text{measured}}/B_{\text{max}}$  vs.  $\sin(\theta)$ , and another is  $(B_{\text{measured}}/B_{\text{max}})/\sin(\theta)$  vs.  $\theta$ . To do this, you will need to adjust your data properly so that the max and min values of  $B_{\text{measured}}$  have the same magnitude. Then you need to locate where  $\theta = 0$  occurs in your data set so that you can establish the correspondence between  $\theta$  and the angles recorded from the divided circle of the Hall probe. If you have questions about how to do this, see the instructor.
2. The magnetic field of a long-straight wire In addition to providing a table containing your data, you are to graph the data in such a way so as to expect a straight-line relationship. One possibility is plotting  $B$  vs.  $\sin(\theta)/r$ , in which case you would expect a straight line with slope equal to  $2.00 \times 10^{26}$  Tm. Now here is a problem we want you to confront. Because  $r$  and  $B$  vary over a range of nearly two orders of magnitude in his experiment, a log-log graph is a MUCH better way of displaying the  $B$  vs.  $r$  results than the more customary linear-linear approach. So please plot the  $B$  vs.  $\sin(\theta)/r$  with log-log graph.
3. The magnetic field of an N-turn circular loop As above, one way to graph your data according to a theoretical straight-line relationship is to plot  $B_z$  vs.  $R^{-1}(1 + z^2/R^2)^{-3/2}$ . Then what should the slope of such a graph be? (As you may have guessed, log-log form is a MUCH better approach, but if one log-log graph per experiment is your limit, you may drop back to the linear-linear approach. Talk over this choice with your instructors if you would like to learn about the pros and cons of the two approaches.) By the way, in figuring out the uncertainty associated with such a complicated function, one perfectly good way is to evaluate the complicated function both with best values and then with uncertainty-adjusted values in  $z$  and  $R$ . The difference gives you the overall uncertainty in that complicated function without having to take these somewhat tedious derivatives. You can also use the curve fitting tool to fit  $B_z$  as a function of  $z$  to get the radius of the circular loop.

4. The magnetic field of a Helmholtz pair of coils For the regions outside the coils, figure out what you can plot  $B$  against in order to hope to obtain a straight line relationship. (Again, we recommend the log-log approach, but if as above you have reached your log-log limit, you may opt for the linear-linear approach in your choice of graphical display.) For the region  $-R < z < 0$  (between the coils), do what the Instructions suggest and plot  $B$  *vs.*  $z$ , both for your experimental results and for the theory, and then plot  $B_{\text{exp}}/B_{\text{theory}}$  *vs.*  $z$  as a good way to illuminate the degree to which experiment and theory correspond to one another.

## 5 Appendix: Review of Relevant Formulas

1. The hall probe is sensitive to the magnetic field perpendicular to the probe cross section

$$B_{\text{PROBE READING}} = |\vec{B}| \sin \theta \quad (1)$$

where  $\theta$  is defined in Fig. 2 below.

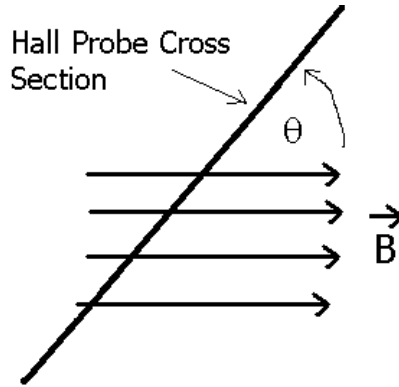


Figure 2: Hall probe geometry.

2. The field due to a wire segment at a distance  $r$  from the central axis of the wire is obtained by integrating the Biot-Savart Law over the length of the segment. The Biot-Savart law is

$$d\vec{B} = \frac{\mu_o}{4\pi} \frac{I d\vec{l} \times \hat{r}}{r^2} \quad (2)$$

The field due to the wire segment is

$$B = \frac{\mu_o I}{2\pi r} \left[ \frac{\sin \theta_1 + \sin \theta_2}{2} \right] \quad (3)$$

where  $\theta_1$  and  $\theta_2$  are as defined in Fig. 3 below.

3. On the axis of a single coil with  $N$  turns, the field is given by

$$B = \frac{\mu_o N I}{2R} \frac{1}{\left(1 + \frac{z^2}{R^2}\right)^{3/2}} \quad (4)$$

Note: a real coil has width. The various quantities in this formula are as defined in Fig. 4 below.

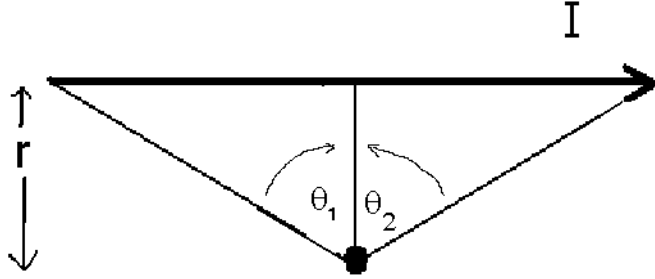


Fig. 3. Geometry of finite Long straight wire.

Figure 3: Geometry of finite long straight wire.

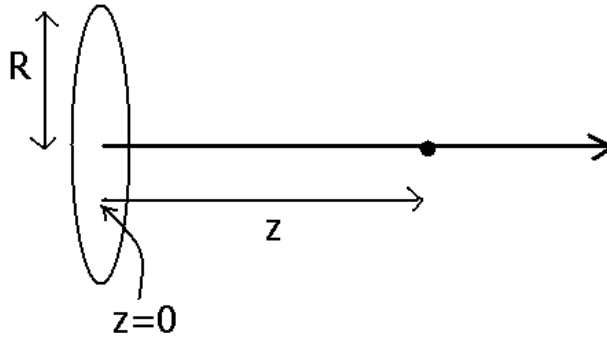


Figure 4: Geometry for a field due to a single very thin coil.

4. On the axis of two coils in the Helmholtz configuration with  $N$  turns each, the field is given by

$$B = \frac{\mu_o NI}{2R} \left[ \frac{1}{\left(1 + \frac{z^2}{R^2}\right)^{3/2}} + \frac{1}{\left(1 + \frac{(R+z)^2}{R^2}\right)^{3/2}} \right] \quad (5)$$

Note: a real coil has width. The various quantities in this formula are as defined in Fig. 5 below. Between the coils  $-R < z < 0$ ; to the right of the coils  $z > 0$ .

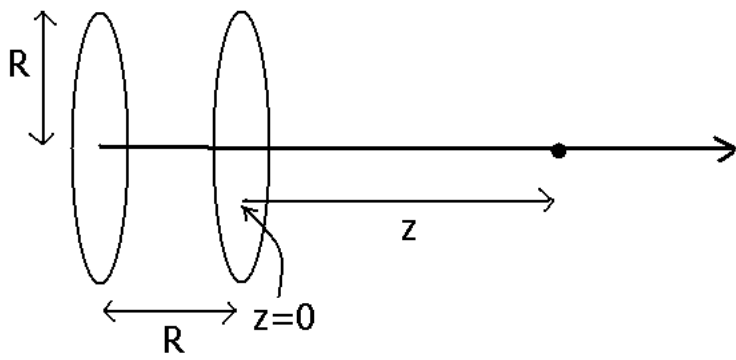


Figure 5: Two coils in Helmholtz configuration, i.e. a distance  $R$  apart.