Uber Rocket (Calculus)
Section S

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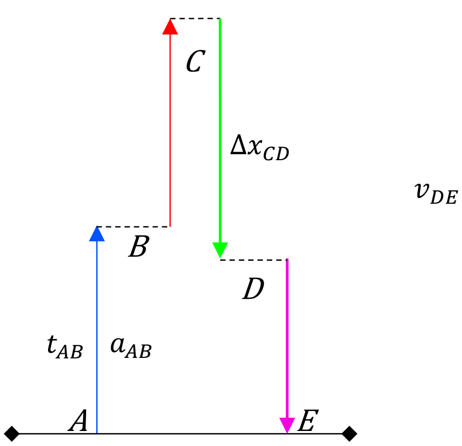
Problem Description

One calm afternoon Calculus Cam decides to launch Hamster Huey into the air using a model rocket. The rocket is launched straight up off the ground, from rest. The rocket engine is designed to burn for specified time while producing non-constant net acceleration given by the equations below. After the engine stops the rocket continues upward in free-fall. A parachute opens after the rocket falls a specified vertical distance from its maximum height. When the parachute opens, assume the rocket instantly stops, and then increases speed to a terminal velocity given by the equation below. Assume the air resistance affects the rocket only during the parachute stage.

Diagram



Givens

$$t_{AB} = 4.1\ (s)$$

$$a_{AB}(t) = -1.2t^2 + 16\ (m/s^2)$$

$$\Delta x_{CD} = -79\ (m)$$

$$v_{DE}(t) = -22(1 - e^{-\frac{t}{6}})\ (m/s)$$

Strategy

To solve this problem, I split the rocket's flight into four parts. The first, AB, is when the rocket is climbing until the engine stops. The second, BC, is from when the engine stops to the max height the rocket reaches. The third, CD is from max height to when the parachute opens. The last, DE, is from when the parachute opens to when it reaches the ground.

In the first stage, AB, you can find the velocity function by taking the indefinite integral of the acceleration function. Using this velocity function, you can find the velocity at point B. Also, by taking the indefinite integral of the velocity function you can get the position function and find the position at point B.

For the second stage, BC, you already know that the velocity at the max height (C) is 0. You also know that the acceleration is from gravity and is -9.8 (m/s^2). You also found out the velocity at point B, which is the initial velocity for this second stage. Using this information, you find the time of the stage, t_{BC} using equation 2. You also find the position at point C using equation 3.

For the third stage, CD, it is given the displacement, so it is easy to calculate the position at point D. You also use this, along with the acceleration and the velocity at point C, 0 (m/s^2) to find the time of the stage, t_{CD} with equation 3.

For the final stage, DE, it is given the velocity function (which is in terms of time). In order to find the time of this stage, t_{DE}, we must set the displacement (negative of the position at D) equal to the definite integral of the given velocity function (v_{DE}) from t=0 to t=t_{DE} and solve for t_{DE}.

Now we must add t_{AB} + t_{BC} + t_{CD} + t_{DE} to get our final amount of time.

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Stage AB

$$\Delta v_{AB}(t) = \int a_{AB}(t)\, dt$$

$$\Delta v_{AB}(t) = \int (-1.2t_{AB}^2 + 16)\, dt$$

$$\Delta v_{AB}(t) = -0.4t_{AB}^3 + 16t_{AB}$$

$$v_B(t) = \Delta v_{AB} + v_A$$

$$v_B(t) = -0.4t_{AB}^3 + 16t_{AB} + 0$$

$$v_B(t) = -0.4t_{AB}^3 + 16t_{AB}$$

$$v_B = -0.4(4.1)^3 + 16(4.1)$$

$$\underline{v_B = 38.032\ (m/s)}$$

$$\Delta x_{AB}(t) = \int v_{AB}(t)\, dt$$

$$\Delta x_{AB}(t) = \int (-0.4t_{AB}^3 + 16t_{AB})\, dt$$

$$\Delta x_{AB}(t) = -0.1t_{AB}^4 + 8t_{AB}^2$$

$$x_B(t) = \Delta x_{AB} + x_A$$

$$x_B(t) = -0.1t_{AB}^4 + 8t_{AB}^2 + 0$$

$$x_B(t) = -0.1t_{AB}^4 + 8t_{AB}^2$$

$$x_B = -0.1(4.1)^4 + 8(4.1)^2$$

$$\underline{x_B = 106.22\ (m)}$$

Stage BC

$$v_C = a_{BC} * t_{BC} + v_B$$

$$0 = (-9.8)t_{BC} + 38.032$$

$$\underline{t_{BC} = 3.881\ (s)}$$

$$\Delta x_{BC} = \frac{1}{2}(a_{BC})t_{BC}^2 + v_B * t_{BC}$$

$$\Delta x_{BC} = \frac{1}{2}(-9.8)(3.881)^2 + 38.032 * (3.881)$$

$$\underline{\Delta x_{BC} = 73.796\ (m)}$$

Stage CD

$$\Delta x_{CD} = \frac{1}{2}(a_{CD})t_{CD}^2 + v_C * t_{CD}$$

$$-79 = \frac{1}{2}(-9.8)t_{CD}^2$$

$$t_{CD}^2 = \frac{-79}{-4.9}$$

$$\underline{t_{CD} = 4.015\ (s)}$$

$$x_D = x_B + \Delta x_{BC} + \Delta x_{CD}$$

$$x_D = 106.22 + 73.80 + (-79)$$

$$\underline{x_D = 101.018\ (m)}$$

Stage DE

$$\Delta x_{DE} = \int_0^{t_{DE}} v_{DE}(t)\, dt$$

$$\Delta x_{DE} = \int_0^{t_{DE}} (-22[1 - e^{-\frac{t}{6}}])\, dt$$

$$x_E - x_D = \int_0^{t_{DE}} (-22[1 - e^{-\frac{t}{6}}])\, dt$$

$$0 - 101.018 = \int_0^{t_{DE}} (-22[1 - e^{-\frac{t}{6}}])\, dt$$

$$-101.018 = \int_0^{t_{DE}} (-22[1 - e^{-\frac{t}{6}}])\, dt$$

SOLVER ON CALCULATOR

$$t_{DE} = \sout{-6.016}, 9.323$$

$$t_{TOT} = t_{AB} + t_{BC} + t_{CD} + t_{DE}$$

$$t_{TOT} = 4.1 + 3.881 + 4.015 + 9.323$$

$$\boxed{t_{TOT} = 21.319}$$

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Data Tables

t (s)	a(t) (m/s²)	v(t) (m/s)	x(t) (m)
0.00	16.00	0.00	0.00
1.00	14.80	15.60	7.90
2.00	11.20	28.80	30.40
3.00	5.20	37.20	63.90
4.00	-3.20	38.40	102.40
4.10	-4.17	38.03	106.22
4.10	-9.80	38.03	106.22
5.00	-9.80	29.21	136.48
6.00	-9.80	19.41	160.79
7.00	-9.80	9.61	175.31
7.98	-9.80	0.00	180.02
7.98	-9.80	0.00	180.02
8.00	-9.80	-0.19	180.02
9.00	-9.80	-9.99	174.93
10.00	-9.80	-19.79	160.04
11.00	-9.80	-29.59	135.35
12.00	-9.80	-39.39	100.87
12.01	-9.80	-39.45	100.63
12.01	-0.50	-19.03	100.63
13.00	-0.42	-19.48	98.91
14.00	-0.36	-19.87	94.08
15.00	-0.30	-20.19	86.62
16.00	-0.25	-20.47	76.92
17.00	-0.22	-20.71	65.34
18.00	-0.18	-20.90	52.15
19.00	-0.15	-21.07	37.62
20.00	-0.13	-21.22	21.93
21.00	-0.11	-21.34	5.28
21.31	-0.11	-21.37	0.00

Graphs: