Uber Pulley (Calculus)
Section S

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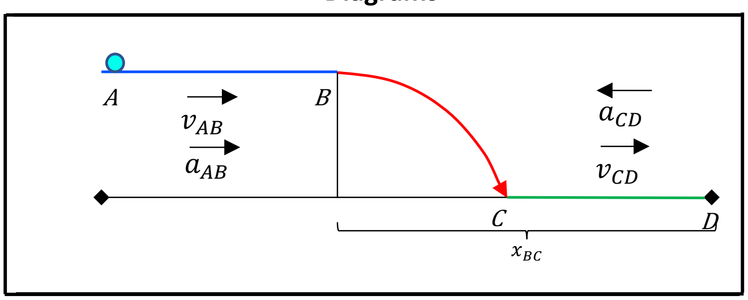
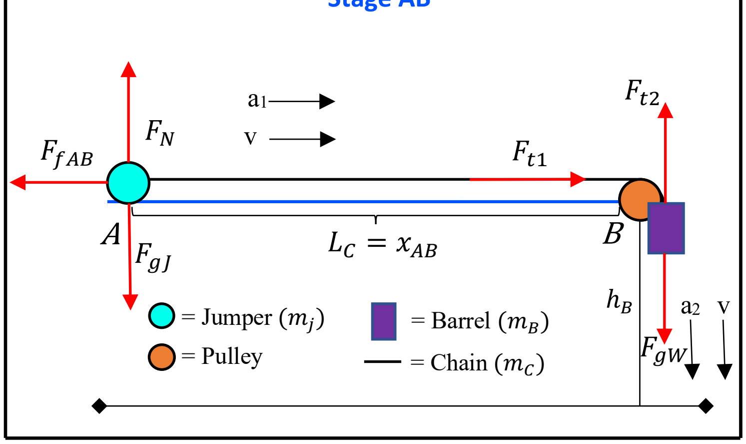
Problem Description

"Jerky" Jerry decided to make a jabberwocky jumper using a pulley system (see diagram). His method was to attach one end of a chain to a barrel of rocks, and the other end to the jumper. He placed the barrel and chain over a massless frictionless pulley, and then walked along a platform away from the pulley to point A (the full length of the chain). When he sat in the jumper he accelerated along the platform to point B and then launched off it while releasing the chain from the jumper and avoiding the pulley. He flew through the air as a projectile to point C, transitioning 75% of his (net) speed into the horizontal direction, and eventually slid to a stop at point D. Note: Ignore any heights of the jumper, pulley, and barrel. Ignore any frictional and normal forces of the chain.

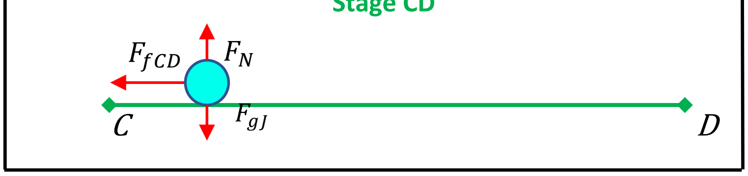
Diagrams



Stage AB



Stage CD



Givens

$$L_C = x_{AB} = 9\ (m)$$
$$m_C = 51\ (kg)$$
$$m_j = 64\ (kg)$$
$$m_B = 153\ (kg)$$
$$h_B = 15\ (m)$$
$$\mu_{AB} = 0.17$$
$$\Delta x_{BD} = 118\ (m)$$

Assumptions

$$F_{t1} = -F_{t2}$$
$$a_1 = a_2$$

Strategy

First, we split up the problem into three stages, stage AB, stage BC, and stage CD.

For Stage AB, we need to find the velocity of Jerry as he lets go of the chain (v_B). So, we must find the acceleration function in terms of the change in position of the chain (Δx_C). In order to find this, we must use a system axis, and find the sum of forces. We need to express the force of gravity on the right side of the pulley (F_{gW}) in terms of Δx_C. We also need to express the mass of the chain on both sides of the pulley (m_{CL} and m_{CR}) in terms of Δx_C. Then we can find the acceleration function in terms of Δx_C. After that, we can use the acceleration function to find v_B which occurs when the chain moves its full length (L_C) to the right.

For Stage BC Jerry is flying as a projectile, with the initial velocity found during Stage AB (v_B). Just before he lands, he transfers 75% of his speed into horizontal speed for the final stage. So, we must find this new horizontal speed (v_{CDI}) by finding the vertical (v_{CY}) and horizontal speed (v_{CX}) as he lands. To do this, we need to find the time that he is in the air (t_{BC}). We also need to find the horizontal distance covered until he lands (Δx_{BC}).

For the final stage, we need to find the coefficient of friction (μ_{CD}). Using the new horizontal speed found in Stage BC (v_{CDI}), and the distance covered (Δx_{CD}), we find the acceleration of this stage (a_{CD})., and solve for the the friction coefficient.

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Stage AB

$$m_{CL}(\Delta x_C) = \frac{L_C - \Delta x_C}{L_C} * m_C$$

$$\underline{m_{CL}(\Delta x_C) = \frac{9 - \Delta x_C}{9} * 51}$$

$$m_{CR}(\Delta x_C) = \frac{0 + \Delta x_C}{L_C} * m_C$$

$$\underline{m_{CR}(\Delta x_C) = \frac{\Delta x_C}{9} * 51}$$

$$\sum F_{SY}: F_N - F_{gJ} = \cancel{m_j * a}$$

$$F_N - 9.8 * m_j = 0$$

$$F_N = 9.8 * 64$$

$$\underline{F_N = 627.2\ N}$$

$$\sum F_{SX}: F_{gW} \cancel{- F_{t2} + F_{t1}} - F_{fAB} = m_s * a$$

$$9.8 * (m_{CR} + m_B) - \mu_{AB} * F_N$$

$$= (m_j + m_B + m_C) * a$$

$$9.8 * \left(\frac{\Delta x_C}{9} * 51 + 153\right) - 0.17 * 627.2$$

$$= (268) * a$$

$$55.5333 * \Delta x_C + 1499.4 - 106.624 = 268 * a$$

$$\underline{a(\Delta x_C) = \frac{55.5333 * \Delta x_C + 1392.79}{268}}$$

$$a(\Delta x_C) * (dx) = v * (dv)$$

$$\int_{\Delta x_0}^{\Delta x_f} a(\Delta x_C) dx = \int_{v_0}^{v_B} v\, dv$$

$$\int_0^9 \left(\frac{55.5333 * \Delta x_C + 1392.79}{268}\right) dx = \int_{v_0}^{v_B} v\, dv$$

$$55.1645 = \frac{v_B^2}{2} - \cancel{\frac{v_0^2}{2}}$$

$$110.329 = v_B^2$$

$$\underline{v_B = 10.504\ (m/s)}$$

Stage BC

$$h_C = \frac{1}{2} a_{BCY} * t_{BC}^2 + v_{BY} * t_{BC} + h_B$$

$$0 = \frac{1}{2} * 9.8 * t_{BC}^2 + 15$$

$$\underline{t_{BC} = 1.7496\ (s)}$$

$$v_{CY} = (a_{BC}) t_{BC} + \cancel{v_{BY}}$$

$$v_{CY} = (-9.8) * 1.7496$$

$$\underline{v_{CY} = -17.146\ (m/s)}$$

$$v_{CX} = \cancel{(a_{BCX}) t_{BC}} + v_{BX}$$

$$\underline{v_{CX} = 10.504\ (m/s)}$$

$$v_{CXF} = \sqrt{v_{CY}^2 + v_{CX}^2}$$

$$S_C = \sqrt{(-17.146)^2 + (10.504)^2}$$

$$\underline{S_C = 20.108\ (m/s)}$$

$$v_{CDI} = \left(\frac{75}{100}\right) * S_C$$

$$\underline{v_{CDI} = 15.081\ (m/s)}$$

$$\Delta x_{BC} = \cancel{\frac{1}{2} a_{BCX} * t_{BC}^2} + v_B * t_{BC}$$

$$\Delta x_{BC} = 10.504 * 1.7496$$

$$\underline{\Delta x_{BC} = 18.38\ (m)}$$

Stage CD

$$\Delta x_{CD} = \Delta x_{BD} - \Delta x_{BC}$$

$$\Delta x_{CD} = 118 - 18.38$$

$$\underline{\Delta x_{CD} = 99.62\ (m)}$$

$$\cancel{v_D^2} = v_{CDI}^2 + 2 * a_{CD} * \Delta x_{BC}$$

$$0 = (15.081)^2 + 2 * a_{CD} * 99.62$$

$$\underline{a_{CD} = -1.141\ (m/s^2)}$$

$$\sum F_{CDX}: -F_f = m_j * a_{CD}$$

$$-(\mu_{CD}) * (F_N) = 64 * -1.141$$

$$-(\mu_{CD}) * (627.2) = -73.055$$

$$\boxed{\mu_{CD} = 0.1165}$$